HD 1761
G6
#230

~~OC 4 '60~~
~~OCT 1 1 '60~~
OCT 1 8 '60

Division of Agricultural Sciences

UNIVERSITY OF CALIFORNIA

ECONOMICS OF FERTILIZATION FOR SELECTED CALIFORNIA CROPS

Harold O. Carter

Gerald W. Dean

Chester O. McCorkle, Jr.

UNIVERSITY OF CALIFORNIA
DAVIS
JUN 27 1960
LIBRARY

CALIFORNIA AGRICULTURAL EXPERIMENT STATION
GIANNINI FOUNDATION OF AGRICULTURAL ECONOMICS

Mimeographed Report No. 230

March 1960

TABLE OF CONTENTS

Page

FOREWORD i

INTRODUCTION 1

OBJECTIVES 5

THEORETICAL FRAMEWORK FOR DETERMINING ECONOMIC OPTIMA. 7

  Physical Fertilizer-Yield Response Models 7

    Yield response to one variable nutrient. 7

    Yield response to two variable nutrients--a production surface . 10

  Economic Interpretation of Yield Response Models. 23

    Optimum use of a single variable nutrient. 23

    Optimum combination of two or more variable nutrients. 25

ECONOMIC FERTILIZATION RATES BASED ON DATA FROM FIELD EXPERIMENTS. . . 30

  Corn. 32

  Potatoes. 43

    Fertilization of high value crops with price uncertainty 46

    Fertilization problems with inter-year variation in response . . 49

  Alfalfa 52

  Sugar Beets 55

  Rice. 57

EXPERIMENTAL DESIGNS FOR GENERATING THE YIELD RESPONSE SURFACE 66

  Experiments for Two or More Nutrients 68

APPENDIX A 71

LIST OF TABLES

Table — Page

1 Tons of Plant Nutrients Used on Selected Crops in California, 1954. 4

2 Purchases of Fertilizer for Use on Farms, Leading Counties in California, 1954. 6

3 Optimum Rates of Nitrogen Fertilizer Application (pounds per acre) on Corn Under Different Price-Cost Situations (Based on 1955 Field Trial in Tulare County) 35

4 Optimum Levels of Nitrogen Fertilization (pounds per acre) for Potatoes Grown in Seven Locations Under Different Price-Cost Situations. 47

5 Comparative Returns from Three Systems of Fertilizing Late Spring Potatoes in Kern County Over the Period 1953-1958. . . 48

6 Optimum Rates of Phosphate (P_2O_5) Application per Acre on Alfalfa in Tulare County (Based on 1957 Experiment on Madera Loam. 54

7 Most Profitable Range of Nitrogen Application for Sugar Beets for Five Areas of California Under Various Price-Cost Situations. 58

8 Nutrient Combinations, Costs and Marginal Rates of Substitution for Selected Levels of Rice Yield 64

9 Example of Data Arrangement for Determining Single Variable Response Curve for Sugar Beets. 68

10 Example of Experimental Arrangement for N and P to Estimate a Response Surface (Figures within squares designate number of replications) 70

LIST OF FIGURES

Figure

1 Annual Tonnage of Fertilizer Used in the U.S. and California. . 2

2 Fertilizer Materials by Types Used in California, 1920-1957 . . 3

3 Form of Yield Response for Single Nutrient with Other Factors Held at Predetermined Levels. 9

4 "Family" of Yield Response Functions with Nutrient X_1 Variable, at Four Levels of Nutrient X_2, Other Factors Held at Predetermined Levels. 11

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

LIST OF FIGURES

Figure Page

5 A Production Surface Illustrating Perfect Substitution Between Two Nutrients 14

6 Yield Contour Map Corresponding to Figure 5 15

7 A Production Surface Illustrating Perfect Complements Between Two Nutrients 17

8 Yield Contour Map Corresponding to Figure 7 18

9 A Production "Surface" Illustrating a Special Case of Perfect Complements Between Two Nutrients 19

10 A Production Surface Illustrating Changing Rates of Substitution Between Nutrients Along the Yield Contours 20

11 Yield Contour Maps Corresponding to Figure 4 with Superimposed Isoclines 21

12 Optimum Application of N_1 with Unlimited Capital. 26

13 Geometric Counterpart of Cost Minimization Equation (6) 28

14 Corn Response to Varying Quantities of Nitrogen Application (based on 1955 field trial in Tulare County). 33

15 Relationship Between the Nitrogen - Net Corn Price Ratio and the Most Profitable Rate of Nitrogen Application on Corn (based on 1955 field trial in Tulare County). 37

16 Relationship Between Nitrogen Application and Percentage Return on the Last Dollar Invested in Nitrogen for Corn, at Specified Price-Cost Ratios 39

17 Corn Response to Nitrogen Fertilizer, Davis, California, 1951 . 41

18 Relationship Between the Nitrogen-Net Corn Price Ratio and the Most Profitable Level of Application (Derived from corn response data in Figure 7). 42

19 Potato Yield Response to Nitrogen, Selected California Areas. . 44

20 Potato Yield Response to Varying Levels of Nitrogen, 11 Years Period 1942-1953 (U.S. Cotton Field Station, Kern County) . . 50

21 Alfalfa Yield Response to Varying Quantities of Phosphorus (Tulare County 1957). 53

22 Sugar Beet Yield Response to Varying Quantities of Nitrogen, Selected California Areas by Years. 56

23 Combinations of Nitrogen and Phosphorus for Studying Yield Response of Rice. 59

24 Yield Isoquants and Isoclines for Selected Price Ratios, for Rice Fertilization Experiment in the Sacramento Valley. . . . 61

FOREWORD

A survey of fertilizer research conducted over a period of years by the California Agricultural Experiment Station alone and in cooperation with Agricultural Extension and other agencies reveals breadth both in the aspects of fertilization considered and in coverage of crops. The yield response to various levels and combinations of nutrients, the influence of time of application on yield and quality, the effects of method of application and placement on yield, and other important questions have been studied for many crops in various locations and under a variety of growing conditions.

Relatively few experiments are available, however, which provide data in the form suitable for comprehensive economic analysis. Hence, empirical examples used in this study are limited to corn, rice, potatoes, sugar beets and alfalfa. Fertilization results were also examined for lettuce, peas, spinach, garlic, tomatoes, onions, cabbage, cantaloups, sweet corn, sweet potatoes, watermelons, celery and carrots. Experiments for these crops have investigated many questions of fertilization, but were not designed specifically to produce data for estimation of complete fertilizer response functions. Fertilization of fruit trees presents particular problems of the carry-over effects of nutrients in the soil and the impact of fertilizer practices on subsequent production. Analysis of fruit tree fertilization was therefore omitted in this preliminary study.

The authors wish to acknowledge the generous assistance and cooperation extended to them by Dr. Oscar Lorenz and Dr. John Lingle of Vegetable Crops, Dr. Duane Mikkelsen, Dr. Dale Smeltzer, Dr. Robert Loomis and

Mr. Jack Hills of the Agronomy Department, and Dr. William Martin of the Soils Department, in making available much of the basic data used in this study. Furthermore, the authors are grateful for the help of many persons in the Agricultural Extension Service and others who participated in the experiments that contributed the basic data. Gordon Fisher was particularly helpful in making preliminary computations and analyses.

This preliminary analysis of the economics of fertilizing selected crops in California has been made possible with the financial assistance of the National Plant Food Institute through their Western Regional Director, Dr. Richard Bahme. His personal interest in this research during its progress is greatly appreciated.

ECONOMICS OF FERTILIZATION FOR SELECTED CALIFORNIA CROPS

Harold O. Carter,[1] Gerald W. Dean[1] and Chester O. McCorkle, Jr.[2]

Commercial fertilizer use has materially increased the output of American agriculture in the past 30 years. Some estimates impute nearly 25 percent of total U.S. agricultural production to fertilizer.[3] Fertilizer use is most intensive in highly commercialized agricultural areas such as California. That heavy rates of application are common practice in California is apparent. The state uses 10 percent of the fertilizer applied in the United States on only 3 percent of the nation's cropland. Between 1920 and 1959 the total tonnage of fertilizer used annually in California rose from 66,000 tons to 2 million tons. This is a 30-fold increase compared with the 3-fold national increase (Figure 1).

The rate of growth in the use of commercial fertilizers and agricultural minerals, the two major components of fertilizer materials, has been about equal (Figure 2). Commercial fertilizers include nitrogen, phosphate and potash while agricultural minerals include such products as gypsum, sulfur, lime, sewage sludge, and similar items. Increased use of agricultural minerals reflects in part the increased intensity of cultivation and accompanying soil, water and plant nutrition problems that have arisen.

In 1954 about 4 1/4 million acres were fertilized in California with one or more of the three major nutrients (Table 1). The tonnage of nitrogen applied was more than double the total amount of phosphate and potash. About 60 percent of the nitrogen was used in the production of

---

[1] Assistant Professors of Agricultural Economics, Assistant Agricultural Economists in the Experiment Station and on the Giannini Foundation, University of California, Davis, California.

[2] Associate Professor of Agricultural Economics, Associate Agricultural Economist in the Experiment Station and on the Giannini Foundation, University of California, Davis, California.

[3] Robertson, L. S., G. L. Johnson and J. F. Davis, "Problems Involved in the Integration of Agronomic and Economic Methodologies in Economic Optima Experiments," in Economic and Technical Analysis of Fertilizer Innovations and Resource Use, edited by E. L. Baum and others (Ames: The Iowa State College Press, 1957), Ch. 20, pp. 226-240.

ECONOMICS OF FERTILIZATION FOR SELECTED CALIFORNIA CROPS

[illegible][1] [illegible][2] [illegible]

Commercial fertilizer use has materially increased the output of American agriculture in the past 20 years. Some estimates credit fully 20 percent of total U.S. agricultural production to fertilizer.[3] California has a more intensive or highly commercialized agriculture than most states, and the [illegible] of fertilization and crops produced in California is apparent. The State used 10 percent of the fertilizer applied in the United States on only 3 percent of the Nation's cropland. Between 1940 and 1959 the total tonnage of fertilizer used annually in California rose from 150,000 tons to 1 million tons. This is a [illegible] increase compared with the [illegible] national increase (figure 1).

The rate of growth in the use of commercial fertilizers, and agricultural minerals, (by far the major component of fertilizer materials), has been about equal (figure 2). Commercial fertilizers include nitrogen, phosphoric acid and potash while agricultural minerals include such materials as gypsum, sulfur, lime, [illegible], and [illegible]. Increased use of agricultural minerals in part [illegible] increased intensity of cultivation and [illegible] and other [illegible] problems than [illegible].

In 1959 about [illegible] million acres were fertilized in California with one or more of the three major nutrients (Table 1). The tonnage of nitrogen applied was more than double the total tonnage of phosphoric acid and potash. About 90 percent of the nitrogen was used in the production of

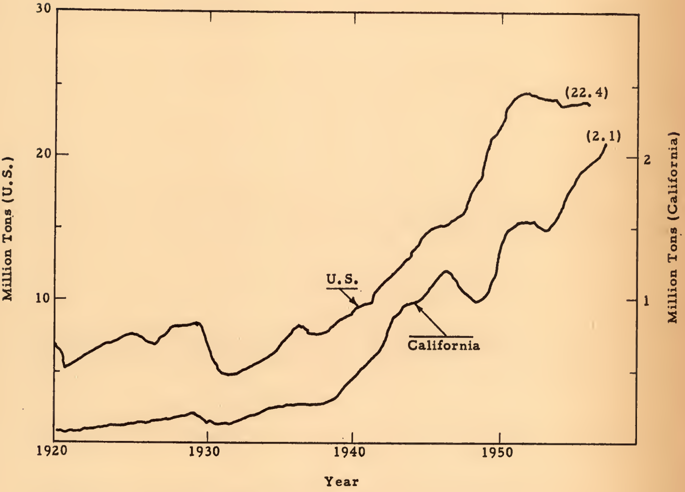
---

[1] Associate Professor of Agricultural Economics, Associate Agricultural Economist in the Experiment Station and on the Giannini Foundation, University of California, Davis, California.

[2] Assistant Professor of Agricultural Economics, Assistant Agricultural Economist in the Experiment Station and on the Giannini Foundation, University of California, Davis, California.

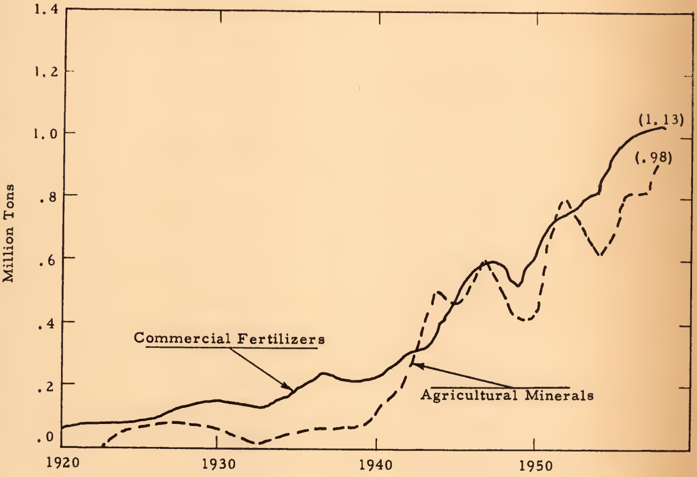
[3] [illegible], "[illegible]," in *Economic and Technical Analysis of Fertilizer Innovations and Resource Use*, edited by E. L. Baum and others (Ames, Iowa: Iowa State College Press, 1957), pp. [illegible].

Figure 1. Annual Tonnage of Fertilizer Used in the U. S. and California



Sources: (1) U.S. Department of Agriculture, Statistical Bulletin 191
(2) U.S. Bureau of the Census, Statistical Abstract of the United States 1959
(3) California Department of Agriculture, Special Publication 269, "Fertilizing Materials, 1957."

Figure 2. Fertilizer Materials by Types Used in California, 1920-1957



Source: California Department of Agriculture, Special Publication 274, "Fertilizer Materials, 1958."

TABLE 1

Tons of Plant Nutrients Used on Selected Crops in California, 1954

Crop	Acres fertilized	Tons of plant nutrients		
		N	P_2O_5	K_2O
	1	2	3	4
Cotton	745,706	30,351	6,101	214
Fruits	706,716	36,022	5,654	5,529
Vegetables	468,493	11,672	10,328	3,398
Sugar beets	207,429	9,989	4,886	522
Corn	127,223	4,023	715	66
Potatoes	90,845	5,670	1,875	576
Sorghum	84,977	1,905	514	397
Barley	586,956	11,733	3,185	192
Rice	376,019	7,640	80	0
Wheat	99,345	1,831	755	25
Hay and pasture	510,591	7,483	24,116	3,922
Other farm use	222,874	5,441	3,687	638
Totals	4,227,174	133,760	61,896	15,479

Source: U.S. Department of Agriculture Statistical Bulletin No. 216, "Fertilizer Used on Crops and Pasture in the United States, 1954 Estimates," USDA, AMS, August 1957.

cotton, fruits and vegetables. These crops also received slightly more than a third of the phosphate and about 60 percent of the potash. About 40 percent of the phosphate was applied to hay and pasture crops.

Ten counties in California accounted for approximately two-thirds of the fertilizer purchases for farm use in 1954 (Table 2). Counties important in cotton, fruit and vegetable production dominate the list; nearly one-third of the fertilizer was used in three counties--Imperial, Kern and Fresno.

OBJECTIVES

The importance of fertilizer use to California agriculture is recognized through the breadth of fertilizer research carried on in the California Agricultural Experiment Station. Fertilizer-yield relationships for field crops and vegetable crops as well as for fruit and nut crops have been studied in recent years. An ultimate purpose of this research is to provide information useful to farmers in determining economic use of fertilizer. With California farmers currently spending in excess of $120 million per year for fertilizers, economic guides to its use are essential to an efficient agriculture. Too little attention has been directed to the economic implications of fertilization. Recommendations to farmers sometimes have been based primarily upon physical yield response data, with little attention to economic factors such as product and fertilizer price relationships, operating capital limitations of farmers and risk and uncertainty. Economic considerations, where introduced, have been confined primarily to a comparison of budgeted costs and returns for two or three selected combinations and levels of fertilizer nutrients. Further analysis of the economics of fertilizer practices on major California crops is needed to provide more complete recommendations.

TABLE 2

Purchases of Fertilizer for Use on Farms, Leading Counties in California, 1954

County	Tons of commercial fertilizer used	Percent of state total
	1	2
Imperial	85,400	10.5
Kern	84,267	10.3
Fresno	81,626	10.0
Monterey	66,132	8.1
Tulare	47,096	5.9
Riverside	37,179	4.6
San Joaquin	32,420	4.0
Santa Clara	27,262	3.3
Kings	26,050	3.2
Los Angeles	24,846	3.0
Total, 10 counties	512,278	62.9

Source: U.S. Bureau of the Census, Census of Agriculture 1954, Vol. III, part 10.

The primary objective of this study is to survey past fertilizer research in California, making economic analysis where possible. Emphasis is placed on principles and methods by which future work might be made more adaptable to economic analysis. The specific objectives of this study are:

1. Review relevant economic principles as they relate to optimum fertilizer use;
2. Inventory fertilizer response data available from past experiments;
3. Demonstrate principles of optimum fertilizer use for crops with adequate response data;
4. Suggest experimental designs which would allow more adequate economic analysis.

THEORETICAL FRAMEWORK FOR DETERMINING ECONOMIC OPTIMA

It is necessary to first develop the logic underlying the economics of fertilizer use. Various physical fertilizer-yield models will then be hypothesized, followed by a discussion of the economic significance of these models for fertilizer recommendations.

Physical Fertilizer-Yield Response Models

Yield response to one variable nutrient.--The simplest response relationship is explored first: Yield per acre is some function of a single variable nutrient with other plant nutrients, soil and moisture conditions, seed variety, etc., fixed at some constant level, as indicated in Equation (1).[1]

---

[1] The vertical bar following X_1 indicates that all variables to the right of this bar are held constant at some predetermined level. Thus, equation 1 is the relationship between yield and varying rates of X_1 (e.g., nitrogen) with all other factors "held constant."

(1) $$Y = f(X_1 \mid X_2, X_3, \ldots X_n, V, S, \ldots, Z) + u$$

Where:

Y = yield (quantity)

X_1 = nutrient 1

X_2 = nutrient 2

X_3 = nutrient 3

X_n = nutrient n

V = seed variety

S = general soil conditions

Z = other practices and inputs

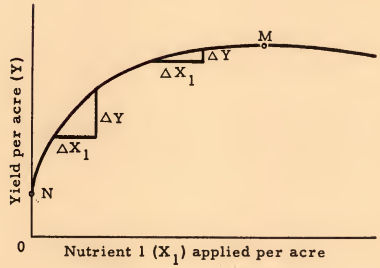
u = uncontrollable or random disturbances

The well-known principle of "diminishing marginal returns" suggests the general form of the response function illustrated in Figure 3: As increased amounts of the variable factor (nutrient 1) are applied to a given land area with other factors held fixed at predetermined levels, output or yield increases at a decreasing rate until a maximum is obtained.[1] In other words, the added yield ($\triangle Y$) from a given increase in nutrient 1 ($\triangle X_1$) declines as higher levels of X_1 are applied. Thus, Figure 3 indicates that the ratio $\frac{\triangle Y}{\triangle X_1}$ declines as more fertilizer is applied. Moreover, a point is reached where additional applications of the factor eventually result in

---

[1] Another form of the curve is the classical case where yield first increases at an increasing rate, then increases at a decreasing rate, and eventually declines. Fertilizer response data seldom show this first stage (yields increasing at increasing rate) perhaps because of the nutrient level in the soil or residual fertilizer present.

Figure 3. Form of Yield Response for Single Nutrient with Other Factors Held at Predetermined Levels



a decline in total yield. Figure 3 shows that maximum production is reached at point M; further application of X_1 lowers total yield per acre. At the zero (applied) level of X_1, a yield on ON indicates that sufficient quantities of X_1 are available in the soil before application (plus the fixed levels of other factors) to allow a nonzero yield.

Other response relationships between Y and X_1, based on higher or lower levels of the other "fixed" nutrients, could also be obtained. Hence, with several levels of fixed factors a "family" of response functions relating X_1 and yield could be established. Figure 4 illustrates a "family" of four yield response curves from variable quantities of nutrient 1 (X_1), with nutrient X_2 held constant at each of four levels (and all other variables held constant).[1] (Points A, B and C in Figure 4 will be explained later.) In the same way, an infinite number of relationships between Y and X_1 could be obtained by setting the "fixed" factors at various levels and combinations of levels.

Yield response to two variable nutrients--a production surface.--At the next level of complexity is the relationship between yield and two variable nutrients, as indicated in Equation 2. Here yield (Y) is a

$$(2) \qquad Y = f(X_1, X_2, \bigg| \; X_3, \ldots X_n, V, S, \ldots, Z) + u$$

function of varying levels of X_1 and X_2 with the other factors "fixed" at some predetermined level. Solid geometry is required to illustrate the

---

[1] Of course, if the horizontal axis in Figure 4 measured the total quantity of nutrient X_1 (both the residual and applied quantities) available to the plant, the "family" of response curves would all start from the origin.

[illegible]

[illegible]

[illegible]

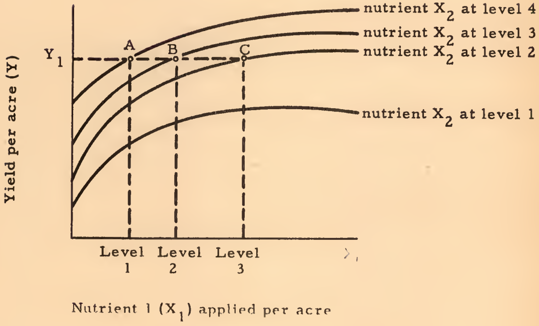
[illegible]

[illegible]

[illegible]

[illegible]

Figure 4. "Family" of Yield Response Functions with Nutrient X_1 Variable, at Four Levels of Nutrient X_2, Other Factors Held at Predetermined Levels.



three-dimensional surfaces relating yield to two variable nutrients. Two extreme cases of hypothetical production surfaces are considered first: (1) where two nutrients are perfect substitutes; (2) where two nutrients are perfect complements. While neither of these extreme surfaces is expected to be found in empirical fertilizer production function analysis, they serve as useful introductions to the type of surface generally obtained.

The term "substitution" as used by the economist may have different connotations for the physical or biological scientist. To the latter, "substitution" implies that one nutrient is "taking the place" of another nutrient in the growth process. Physiologically the nutrient does not substitute for another in the chemical process of the plant; clearly nitrogen cannot be "converted" into phosphate by a chemical reaction. The economist's meaning of "substitution" is based on the hypothesis that specified yield levels can be attained with several different combinations of nutrients. This concept can be illustrated by reference to the "family" of response curves in Figure 4. Yield level Y_1 can be attained at point A (level 1 of nutrient 1 and level 4 of nutrient 2), point B (level 2 of nutrient 1 and level 3 of nutrient 2) or point C (level 3 of nutrient 1 and level 2 of nutrient 2). In essence, points A, B and C represent three "substitute" ways of attaining yield level Y_1. The economic importance to the farmer of this "substitution" is readily apparent: If two or more combinations of nutrients will produce the same yield, the cheapest combination should be used. It is in this economic context that "substitution" relationships are discussed for the remainder of the study.1/

1/ Figure 4 illustrates only one possible "family" of response curves. In some cases, the yield response to additions of one nutrient (e.g., phosphate) is the same for all levels of a second nutrient (e.g., nitrogen). In other cases, the response to (Footnote continued on the next page.)

[illegible]

[illegible]

[illegible]

Figure 5 represents the extreme case where in an economic sense, two nutrients are said to be perfect substitutes. The lines ab, cd, ef, and gh in Figure 5 are yield "contour lines" on a three-dimensional production surface. Each yield contour denotes some constant level of yield attainable through application of various combinations of nutrients 1 and 2. These relationships are clarified by converting the three-dimensional diagram (Figure 5) to a two-dimensional diagram (Figure 6). The contours are labeled as to yield level much as contour lines are labeled as to elevation on a topographic map. The yield contour ab in Figure 6 means that a yield of 100 can be attained with Oa of nutrient 1 and none of nutrient 2; Ob of nutrient 2 and none of nutrient 1; or any combination of nutrients 1 and 2 falling on line ab. In an economic sense, Ob of nutrient 2 can be said to "replace" or "substitute" for Oa of nutrient 1 in attaining a yield of 100. Thus, the economic "substitution rate" between nutrients 1 and 2 in this case is $\frac{Oa}{Ob}$ --the slope of the yield contour. Generalizing, the substitution rate between two nutrients is defined as the slope of the yield contour at a specified point on that contour. Since the yield contours in Figure 6 are straight lines, the substitution rate is the same at all points along that contour. Nutrients which substitute at the same rate for one another regardless of the proportions in which they are combined are called "perfect substitutes."

1/ (continued from pervious page) (say) phosphate is a function of the level of nitrogen; i.e., "interaction" is said to be present. One case of "interaction" may be explained as follows: With both N and P at low levels, an addition of P alone may promote increased root growth and allow N to be "extracted" from soil previously not explored by roots. In this case, an addition of P makes more of both P and N available to the plant. However, with N at a high level and P still at a low level, additions of P may cause lesser yield response; the plant has been "flooded" with N so that expanded root growth from P does not make more N available to the plant. These examples illustrate "interaction"--yield response to P depends on the level of N. It is emphasized, however, that interaction is not necessary for the "economic substitution" between nutrients discussed in the text. Interaction only changes the nature of this substitution.

[illegible]

[illegible]

[illegible]

Figure 5. A Production Surface Illustrating Perfect Substitution Between Two Nutrients

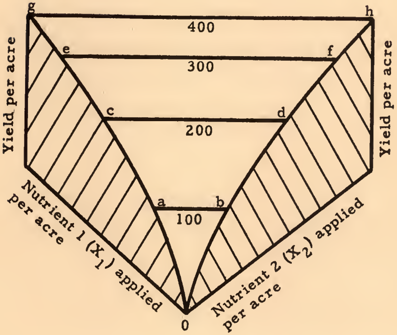
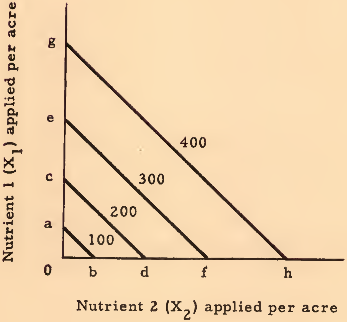

g
h
400
e
f
300
c
d
200
a
b
100
0
Yield per acre
Yield per acre
Nutrient 1 (X_1) applied per acre
Nutrient 2 (X_2) applied per acre

Figure 6. Yield Contour Map Corresponding to Figure 5.


Nutrient 1 (X_1) applied per acre
g
e
c
a
0
b
d
f
h
100
200
300
400
Nutrient 2 (X_2) applied per acre

The production surface depicted in Figures 5 and 6 is sufficiently flexible to allow diminishing returns to fertilizer similar to Figure 3. Diminishing returns are illustrated by yield contours increasingly farther apart as higher yield levels are reached (Figure 6). For example, to double output (e.g., from 100 to 200) requires more than twice the nutrient inputs (e.g., Oc of X_1 is greater than twice Oa of X_1; Od of X_2 is greater than twice Ob of X_2).

The other extreme production surface is that of perfect complements illustrated in Figures 7 and 8. Here one nutrient must be accompanied by a certain minimum amount of the second nutrient in order to attain a particular level of yield. The yield contour lines in Figure 8 indicate, from the slopes, that the substitution rate between nutrients is either zero or infinity. The "corner" of each yield contour specifies the minimum quantity of each nutrient necessary for producing that level of yield; from this point additions of either nutrient alone would be wasted since no additional yield is forthcoming. In other words, there is no range of substitution between nutrients in this case. This model is similar to hypotheses advanced by soil chemists von Liebig, Meyer and Wollney. They believed that fertility nutrients must be combined in fixed proportions in order to attain any positive yield. The surface is then represented by a "knife's" edge traced from the loci of points represented by the "corners" of the contours in Figure 8. This special case of perfect complements is illustrated in Figure 9.

A more realistic fertilizer response surface than those shown in Figures 5 through 9 is indicated by Figures 10 and 11. Figures 10 and 11 show diminishing returns to each nutrient alone or for nutrients combined

Figure 7. A Production Surface Illustrating Perfect Complements Between Two Nutrients

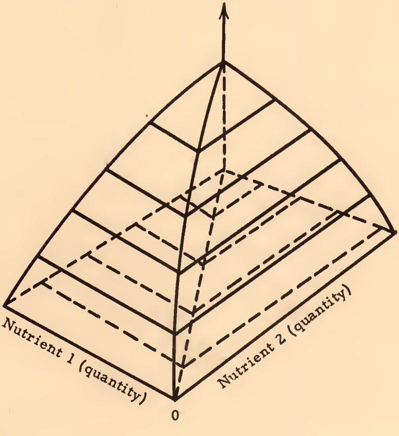


Figure 8. Yield Contour Map Corresponding to Figure 7.

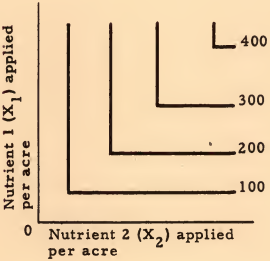


Figure 9. A Production "Surface" Illustrating a Special Case of Perfect Complements Between Two Nutrients

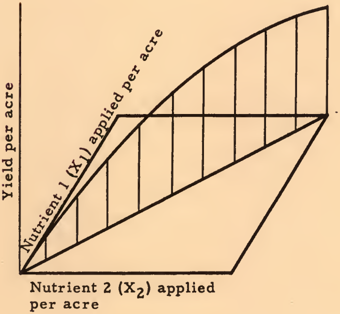

Yield per acre
Nutrient 1 (X_1) applied per acre
Nutrient 2 (X_2) applied per acre

Figure 10. A Production Surface Illustrating Changing Rates of Substitution Between Nutrients Along the Yield Contours

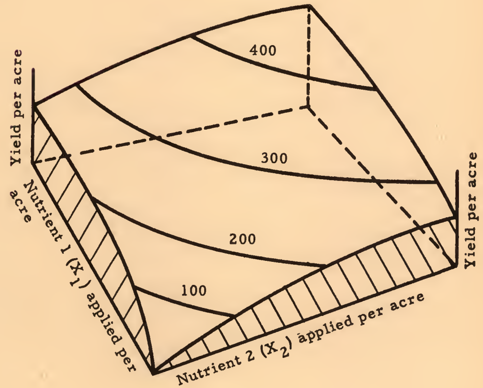
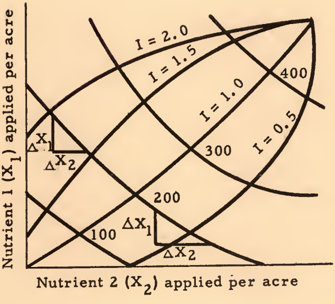

Yield per acre
400
300
200
100
Nutrient 1 (X_1) applied per acre
Nutrient 2 (X_2) applied per acre
Yield per acre

Figure 11. Yield Contour Maps Corresponding to Figure 4 with Superimposed Isoclines



in fixed combinations. Furthermore, the yield contours are all curved and convex to the origin indicating that one nutrient substitutes for the other at a diminishing rate in producing a given yield. That is, for any given yield level, increasing amounts of the added nutrient is required to offset the nutrient being replaced. For example, on the 200 yield contour (Figure 11), the ratio $\frac{\Delta X_1}{\Delta X_2}$ (the substitution rate) declines as more of nutrient 2 is "substituted" for nutrient 1. Figures 10 and 11 also show that the lower yield levels (100 and 200) can be attained with some level of either nutrient alone or with some combination of the two nutrients. However, higher yield levels (300 and 400) require some minimum amount of each nutrient, indicating a degree of complementarity in the production process. Also, in Figures 10 and 11, the range of substitution is narrowed at higher yield levels, until finally maximum yield is characterized by a single point (M), attainable with only one combination of nutrients. Alternatively, maximum yield might be a plateau rather than a point, obtainable with a wide range of nutrient combinations.

The slope of any curved yield contour (e.g., Figure 11) changes with the combination of nutrients considered. This slope has been defined as the substitution rate of one nutrient for the other or more technically as the marginal rate of substitution.1/ A line connecting points of equal slope on successive yield contours is referred to as an isocline. Four different isoclines are presented in Figure 11 (labeled I_1, I_2, I_3 and I_4).

1/ The term marginal rate of substitution is well established in economic literature and therefore is used throughout this manuscript. However, the reader is reminded that this term does not imply a chemical substitution among nutrients, as explained above.

in fixed combinations. Furthermore, the yield contours are all convex to [illegible] [illegible] [illegible] yield [illegible] [illegible] [illegible] level, [illegible] [illegible] [illegible] [illegible] [illegible] (Figure 11), [illegible] $\frac{dX}{dY}$ [illegible] [illegible] Figures 10 and 11 [illegible] [illegible] with a wide range of nutrient combinations.

The slopes of [illegible] yield contours (e.g., Figure 11) [illegible] [illegible] marginal rate of substitution.[1] [illegible] [illegible]

---

[1] The [illegible] marginal rate of substitution [illegible] [illegible] explained elsewhere.

Each isocline represents a constant substitution rate between nutrient 1 and nutrient 2. For example, I_1 denotes an isocline with a slope of 2.0 (i.e., one unit of nutrient 2 replaces two units of nutrient 1). A complete family of isoclines varying in slope from zero to infinity could be derived, each indicating a common slope or substitution rate over the entire surface. These isoclines have direct economic implications discussed later.

Fertilizer response functions have been discussed only for the cases of one and two nutrients variable, other factors held fixed at predetermined levels. However, the concepts introduced can be generalized to cases with three or more nutrients variable. Since graphic presentation with more than three dimensions is impractical, these more complex cases must be handled algebraically.

Economic Interpretation of Yield Response Models

Economic fertilizer recommendations stem from the physical relationships outlined above. It will be shown that in the simplest case, fertilizer recommendations are dependent upon three factors: (1) the physical fertilizer-yield response function, (2) the price of the product and (3) the price of the nutrients.[1/]

Optimum use of a single variable nutrient.--The economic criterion for the most profitable use of nutrient 1 (X_1) with other factors fixed is given in Equation (3).

$$(3) \quad \frac{\triangle Y}{\triangle X_1} = \frac{P_{X_1}}{P_Y} \text{ or } \triangle Y P_Y = \triangle X_1 P_{X_1}$$

[1/] Consideration is also given later to costs of harvest if they vary with level of yield.

The ratio $\frac{\Delta Y}{\Delta X_1}$ is the change in yield associated with a one unit change in nutrient 1, or technically, the marginal product of nutrient 1;[1] P_{X_1} = price per unit of X_1 and P_Y = price per unit of product. Equation (3) states that the economic optimum rate of nutrient 1 is attained when the marginal product of X_1 is equal to the nutrient-crop price ratio. Stating Equation (3) alternatively, optimum fertilizer use occurs when the value of the added product is exactly equal to the cost of the added nutrient ($\Delta YP_Y = \Delta X_1 P_{X_1}$). Nonoptimum rates of use of nutrient 1 are stated in Equations (4) and (5).[2]

$$(4) \quad \frac{\Delta Y}{\Delta X_1} > \frac{P_{X_1}}{P_Y} \qquad \text{or} \qquad \Delta YP_Y > \Delta X_1 P_{X_1}$$

$$(5) \quad \frac{\Delta Y}{\Delta X_1} < \frac{P_{X_1}}{P_Y} \qquad \text{or} \qquad \Delta YP_Y < \Delta X_1 P_{X_1}$$

If the marginal product of X_1 is greater than the price ratio, as in Equation (4), additional profits could be made by adding more X_1; i.e., added returns (ΔYP_Y) are greater than added cost ($\Delta X_1 P_{X_1}$) and higher levels of X_1 could be profitably applied. The converse is illustrated in Equation (5); added returns (ΔYP_Y) are less than added cost (ΔXP_{X_1}) indicating that fertilization is higher than the optimum rate.

---

[1] If a continuous response function is the basis for recommendations, then $\underset{\Delta X_1 \longrightarrow 0}{\Delta Y / \Delta X_1} = \frac{dY}{dX_1}$ and the economic criterion for most profitable fertilizer use becomes $\frac{dY}{dX_1} = \frac{P_{X_1}}{P_Y}$.

[2] The economic optimum $\frac{\Delta Y}{\Delta X_1} = \frac{P_{X_1}}{P_Y}$ assumes that the farmer has unlimited capital to spend for fertilizer. In this case, the last dollar invested in fertilizer is just "paying for itself." If the farmer is limited on capital or has other more profitable investment opportunities, the most profitable rate would be less than indicated by the criterion $\frac{\Delta Y}{\Delta X_1} = \frac{P_{X_1}}{P_Y}$. This question is considered in detail later.

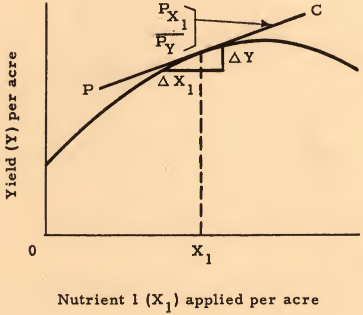
The geometric counterpart of Equation (3) is illustrated in Figure 12. The response curve is identical to that shown in Figure 3. The slope of the curve is $\Delta Y / \Delta X_1$. The line PC represents the price ratio P_{X_1} / P_{Y_1}. Where the price ratio is tangent to the response curve $(\frac{\Delta Y}{\Delta X_1} = \frac{P_{X_1}}{P_Y})$, the optimum condition is realized; X_1' is the resultant optimum usage of nutrient 1. To the right of X_1' is the nonoptimum condition of Equation (5), while to the left of X_1' is the nonoptimum condition of Equation (4). Obviously, the optimum fertilization rate is dependent on the fertilizer-product price ratio: when fertilizer prices are low relative to the product prices, higher rates are profitable, and vice versa.

The above discussion illustrates that economic recommendations require (1) response functions for different types of fertilizer on different crops and for various typical "fixed" levels of the other variables (including moisture conditions, temperature, etc.) and (2) fertilizer and product prices. Recommendations should be sufficiently complete to permit an individual farmer to select alternative fertilization rates consistent with his price expectations and alternative investment opportunities. Since conditions vary considerably between years and among farmers, it is apparent that no "absolute" or "standard" recommendations can be made to cover all situations.

Optimum combination of two or more variable nutrients.--The economic criterion for the optimum combination of two nutrients for a specified level of yield is given in Equation (6). This criterion allows determination

$$(6) \qquad \frac{\Delta X_1}{\Delta X_2} = \frac{P_{X_2}}{P_{X_1}} \qquad \text{or} \qquad \Delta X_1 P_{X_1} = \Delta X_2 P_{X_2}$$

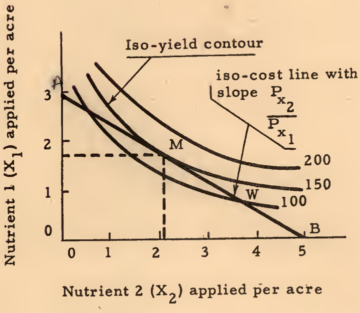
Figure 12. Optimum Application of N_1 with Unlimited Capital



of that particular combination of nutrients 1 and 2 which produces a given level of a yield at minimum cost. P_{X_1} and P_{X_2} refer to the prices of nutrient 1 and 2, respectively, The ratio $\frac{\Delta X_1}{\Delta X_2}$ (marginal rate of substitution) refers to the amount of nutrient 2 required to replace nutrient 1 if yield is to be maintained at a given level. Graphically, $\frac{\Delta X_1}{\Delta X_2}$ is the slope at any point on the yield contour. The minimum cost combination, stated in Equation (6), is where the cost of the nutrient added ($\Delta X_2 P_{X_2}$) is just equal to the cost of the nutrient replaced ($\Delta X_1 P_{X_1}$). Geometrically, the least-cost combination of X_1 and X_2 for a yield of 150 (Figure 13) is shown as the tangency of the 150 yield contour and the price-ratio line AB. Line AB also indicates all possible combinations of X_1 and X_2 that could be purchased with a given expenditure. Assume an expenditure of \$15.00 with the price of nutrient 1 = \$5.00 per unit and the price of nutrient 2 = \$3.00 per unit. Thus, 3 units of X_1 alone, 5 units of X_2 alone, or some combination of X_1 and X_2 falling on line AB can be purchased with \$1.50.[1/] The question is: With an expenditure of \$15.00, what combination of X_1 and X_2 permits the greatest yield? Point M specifies this combination, allowing a yield of 150. Any other combination of X_1 and X_2 on isocost line AB (such as W) provides a lower yield. Stated conversely, point M is the least-cost combination of X_1 and X_2 to produce a yield of 150.

[1/] Note that the slope of the isocost line AB ($^3/_5$) is the inverse price ratio $\frac{P_{X_2}}{P_{X_1}} = \frac{\$0.30}{\$0.50} = {}^3/_5$.

Figure 13. Geometric Counterpart of Cost Minimization Equation (6)



Economic interpretation can now be extended to the various production surfaces hypothesized earlier. Figures 5 and 6 illustrate the special case of perfect substitution between nutrients. Here the slopes of the yield contours are constant; hence, with a given price ratio the minimum cost position will include all of one nutrient or all of the other rather than a combination. For example, if two units of 3-12-12 fertilizer (X_1) substitute for 1 unit of 6-24-24 (X_2) and the price of 6-24-24 is three times that of 3-12-12, fertilizer costs are minimized by using all of 3-12-12. Conversely, if the price of 6-24-24 was less than twice the price of 3-12-12, the least-cost purchase is all 6-24-24.

Perfect complementarity between nutrients is illustrated in Figures 7 and 8. Here price ratios are unnecessary to determine the optimum combination of X_1 and X_2. The point where the slope of the yield contour changes from zero to ∞ (the "corner" of the contour) defines the minimum amounts of each nutrient required to attain that yield; additional amounts of either nutrient would be wasted since no increase in yield would be forthcoming. Hence, in the case of perfect complements, yield nutrients must be combined in a specified ratio regardless of price.

The production surface shown in Figures 10 and 11 illustrates the more common case where the substitution rate between nutrients ($\frac{\Delta X_1}{\Delta X_2}$) changes along the yield contours. In this case the price relationship between nutrients affects the least-cost combination. Too, this minimum-cost combination of nutrients changes with the level of yield if the isoclines are nonlinear.

ECONOMIC FERTILIZATION RATES BASED ON DATA FROM FIELD EXPERIMENTS

A survey of past fertilizer work in California reveals relatively few experiments which provide data completely adaptable to the type of economic analysis outlined above. In general, yield response to more levels and combinations of nutrients is needed for complete economic analysis. Crops with data at least partially suited for economic analysis are corn, rice, potatoes, sugar beets, alfalfa, cotton and small grains.1/ While considerable experimental data are available for other crops, particularly vegetable crops, much of the research has been designed to answer questions other than optimum fertilization levels. For example, such important questions as determining best varieties, seeding rates and depth, fertilizer placement in relation to the seed or root zone, etc., have been examined in some detail. Even for crops analyzed in this report, data often are limited to experiments in selected areas and years and generalization of results to other years and areas may be questionable. While these data limitations attach to the specific empirical results, the economic principles demonstrated are appropriate for a wide range of fertilization problems. The following material is therefore partly illustrative as well as contributing to specific economic recommendations.

1/ An analysis of fertilizer responses in small grains is contained in "Grain Fertilization in California," by W. E. Martin and D. S. Mikkelsen to be published by the California Agricultural Experiment Station. Cotton yield response to nitrogen application for various soils in California has been analyzed in "Nitrogen More Than Pays its Way in California Cotton," by C. O. McCorkle, Jr. and D. S. Mikkelsen in National Fertilizer Review, Jan.-Feb.-Mar. 1954. Therefore, cotton and small grain fertilization have not been included in this study.

The economic analyses of field experiments presented below are designed to illustrate important economic problems encountered in fertilizing various types of crops in California. The simplest problem--determining optimum rates of application of a single nutrient--is illustrated with corn. Corn fertilization is also used to illustrate a method of providing economic fertilizer recommendations to farmers with limited capital.

Analysis of available potato fertilization data allows illustration of two important fertilization problems of particular California crops: (1) the problem of fertilization of crops with a high per acre value, particularly under conditions of extreme price uncertainty; and (2) the problem of fertilizer use when the response function exhibits shifts between years due to weather and other largely uncontrollable variables.

The impact of fertilization upon product quality is illustrated in the analysis of sugar beet response data. Returns to beet growers are affected by both yield and sugar content. However, sugar content depends on the level of nitrogen applied. The approach developed to estimate optimum economic levels of fertilization for sugar beets is applicable to any crop where quality is affected by nutrient level. Selection of optimum combinations of two nutrients is illustrated with field experimental data for rice fertilized with both nitrogen and phosphorus.

The theoretical concepts discussed above have been presented primarily in geometric form. However, in analyzing the empirical results it is necessary to extend the graphic presentation into corresponding algebraic formulations.

In analyzing the economics of fertilization of particular crops, certain methodological problems are encountered. These problems are handled as an integral part of the analysis of that crop, although many of these problems are quite general--for example, capital limitations imposed on farmers and risk and uncertainty.

Corn

The economic analysis for corn is illustrated using data from a trial in Tulare County in 1955.[1] Five alternative levels of ammonia sulfate (0, 50, 100, 150 and 200 pounds of actual nitrogen per acre, replicated four times) were applied and resulting yields per acre measured. Regression Equation (7) represents the yield response to nitrogen derived. The nitrogen-corn yield relationship of Equation (7) is plotted in Figure 14.

$$(7) \qquad \hat{C} = 2988 + 51.47N - 0.1520N^2$$

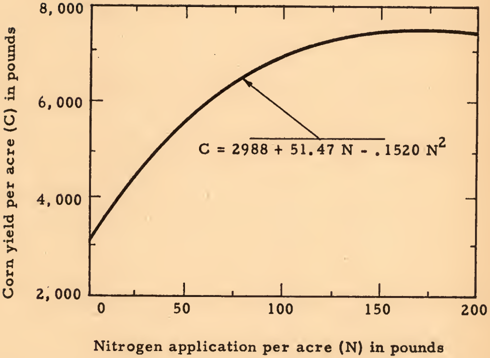
$\hat{C}$ = estimated corn yield in pounds per acre and N = level of nitrogen application in pounds per acre. The negative sign for N^2 indicates diminishing marginal returns to nitrogen; i.e., added yield per pound of added fertilizer diminishes with higher fertilization rates.

Employing the logic developed previously, the economic optimum level of nitrogen fertilization is found by setting the marginal product

---

[1] Unpublished data from field trials conducted by W. E. Pendery, Farm Advisor (Tulare County), Agricultural Extension Service, University of California.

Figure 14. Corn Response to Varying Quantities of Nitrogen Application (based on 1955 field trial in Tulare County)



of nitrogen ($\frac{\Delta \hat{C}}{\Delta N}$ or $\frac{\partial \hat{C}}{\partial N}$) equal to the inverse price ratio ($\frac{P_N}{P_C - SH}$), where P_N = price per 100 pounds of nitrogen, P_C = price per 100 pounds of corn and SH = shelling and hauling costs per 100 pounds of corn.[1] Harvesting costs of corn are not included because they are commonly priced on a per acre basis rather than yield. Equation (8) is the marginal product of nitrogen, derived from Equation (7) as the first derivative of C with respect to N ($\frac{d\hat{C}}{dN}$). The marginal product is equated with the inverse price ratio in Equation (9) and solved for N in Equation (10).

$$(8) \qquad \frac{d\hat{C}}{dN} = 51.47 - 0.3040N$$

$$(9) \qquad 51.47 - 0.3040N = \frac{P_N}{P_C - SH}$$

$$(10) \qquad N = 169.30 - 3.28\,\frac{P_N}{P_C - SH}$$

The economic optimum rates of nitrogen fertilization on corn under various price-cost relationships are presented in Table 3. These data indicate that as nitrogen becomes cheaper relative to corn, the most

---

[1] The equation $\frac{\partial C}{\partial N} = \frac{P_N}{P_C - SH}$ is derived from the profit equation (a), where π = profit and the other variables are defined in the text. The level of nitrogen use to maximize profit is determined by taking the first derivative of the profit equation (a) with respect to nitrogen and setting the result equal to zero as shown in equation (b), and then solving as in equation (c).

$$(a) \qquad \pi = CP_C - CSH - NP_N$$

$$(b) \qquad \frac{\partial \pi}{\partial N} = \frac{\partial C}{\partial N}(P_C - SH) - P_N = 0$$

$$(c) \qquad \frac{\partial C}{\partial N} = \frac{P_N}{P_C - SH}$$

TABLE 3

Optimum Rates of Nitrogen Fertilizer Application (pounds per acre) on Corn Under Different Price-Cost Situations (Based on 1955 Field Trial in Tulare County)[a]

Net corn price (P_C-SH) (dollars per cwt.)	Cost of nitrogen fertilizer applied (P_N) (dollars per cwt.)					
	$6	$8	$10	$12	$14	$16
	1	2	3	4	5	6
	pounds per acre					
$1.60	157	153	149	145	141	136
$1.80	158	155	151	147	144	140
$2.00	159	156	153	150	146	143
$2.20	160	157	154	151	148	145
$2.40	161	158	156	153	150	147
$2.60	162	159	157	154	152	149
$2.80	162	160	158	155	153	151
$3.00	163	161	158	156	154	152

[a] Derived from Equation 10.

profitable rate of applying nitrogen increases. For example, with corn at \$2.05 per hundredweight, shelling and hauling cost at \$0.25 per hundredweight (the net corn price $P_C - SH = \$1.80$) and nitrogen at \$16 per hundredweight the optimum nitrogen rate is 140 pounds per acre. However, when nitrogen drops to \$6, the optimum fertilization rate increases to 158 pounds of N per acre. It follows that an increase in the net price of corn (with the cost of nitrogen held constant) also increases optimum fertilization rates (Table 3). Table 3 also indicates that only price ratios, not absolute prices, are needed to make optimum fertilizer recommendations. For example, the optimum level of nitrogen fertilization remains at 150 pounds when the nitrogen-net corn price ratio $(\frac{P_N}{P_C - SH})$ is 6.0, regardless of whether $P_N = \$12$ and $P_C - SH = \$2.00$ or $P_N = \$15$ and $P_C - SH = \$2.50$. This characteristic can be used to advantage in making recommendations to farmers as shown in Figure 15. Knowing the relevant price ratio, the farmer can "read off" the optimum fertilization rate directly from the graph.

Under the normal range of corn-nitrogen price relationships, this experiment indicates that the recommended amount of nitrogen for corn is about 150 pounds. Even with extreme price relationships the recommended nitrogen application deviates only about 15 pounds from the 150-pound level. However, these recommendations are based on a single experiment which does not represent all corn response relationships.

The above discussion views fertilization as an isolated production practice. However, fertilizer use is more correctly considered in the context of the entire farm, where fertilization is only one of many practices and alternative investment opportunities available. The optimum rates of Table 3 assume sufficient capital available to increase fertilization rates to the point where added cost equals added return. This point corresponds to a zero percent return on the last dollar (marginal dollar)

Figure 15. Relationship Between the Nitrogen - Net Corn Price Ratio and the Most Profitable Rate of Nitrogen Application on Corn (based on 1955 field trial in Tulare County)

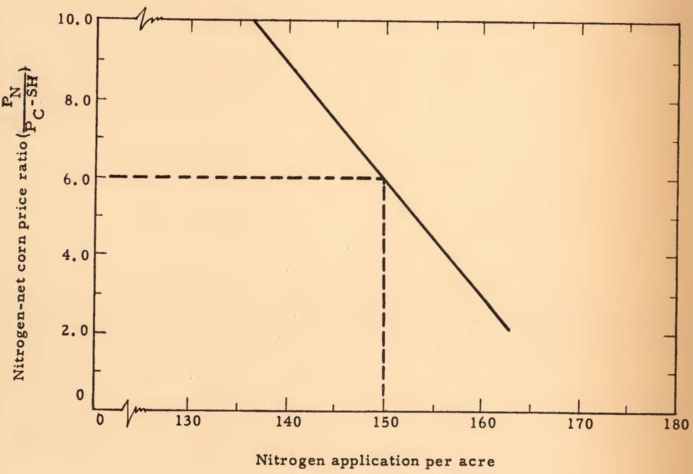
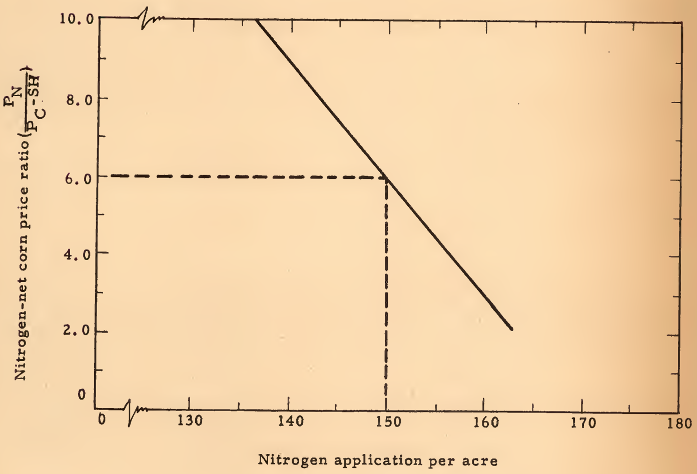


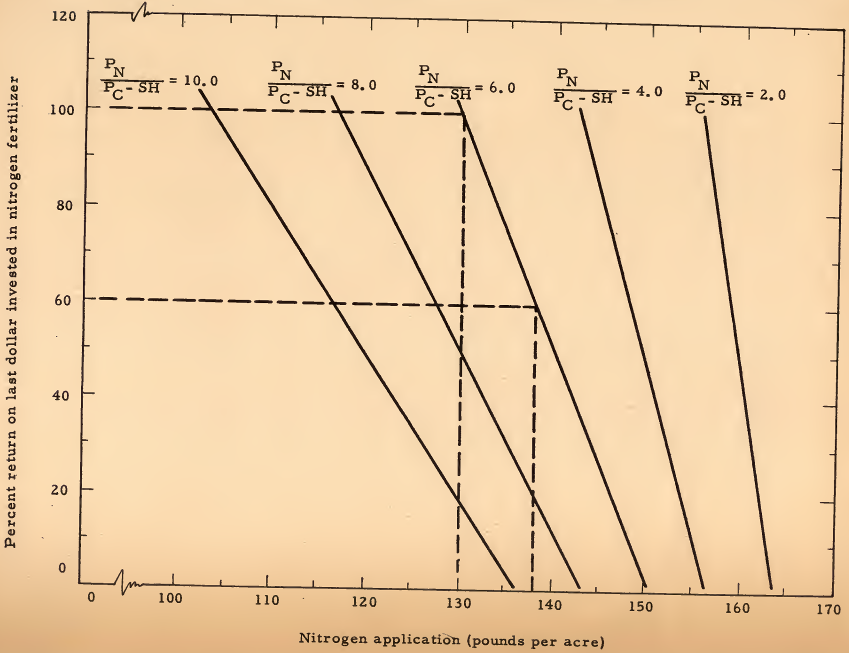
Figure 15. Relationship Between the Nitrogen - Net Corn Price Ratio and the Most Profitable Rate of Nitrogen Application on Corn (based on 1955 field trial in Tulare County)



invested in fertilizer. Obviously, however, if the farmer has available alternative investments promising higher returns, these should be exploited first. A farmer with an opportunity to earn a 50 percent return on additional seasonal labor, or even a 10 percent return on additional common stock, should exploit these alternatives before fertilizing at the "optimum" rates of Table 3. The principle allowing maximum returns to the farm is to equate the marginal productivity of capital (percentage return on the last dollar invested) in all available lines of investment. Fertilizer recommendations providing estimates of the earning power of the last dollar invested in fertilizer would allow approximate equating of marginal returns from fertilizer with returns from other investments. Figure 16 permits estimation of the fertilizer rate necessary to provide a specified return on the "marginal dollar" investment in fertilizer. Thus, with a nitrogen-net corn price ratio of 6.0 (e.g., $P_N = \$12$, $P_C - SH = \$2$), a rate of 150 pounds of nitrogen per acre provides a zero percent return on the marginal dollar. (This recommendation coincides with the "optimum" rate specified in Table 3 and Figure 15 for this price-cost situation and assuming unlimited capital.) However, a farmer with alternative uses of capital earning a 60 percent marginal return should cut nitrogen application to 138 pounds (Figure 16); with alternative uses earning a 100 percent marginal return, nitrogen application should be cut to 130 pounds.1/

1/ A simple method of deriving the optimum fertilizer level when a specified rate of return (r) is desired, is to let the optimizing equation take the form: $\frac{d\hat{C}}{dN} = \frac{(1+r)P_N}{P_C}$.

Figure 16. Relationship Between Nitrogen Application and Percentage Return on the Last Dollar Invested in Nitrogen for Corn, at Specified Price-Cost Ratios



The above discussion illustrates that no standard fertilization recommendation can be made to all farmers--recommendations must be "tailor-made" to fit individual farm situations. Figure 16 provides one method of making recommendations which can be adapted by individual farmers depending on the price-cost situation and their alternative uses of capital.

Results of a second corn fertilization experiment at Davis with 5 levels of nitrogen (0#, 50#, 100#, 150# and 200#) are summarized in Figure 17.[1/] Here yield response to nitrogen is nearly linear up to a 150-pounds application; yield then levels off sharply to added increments of fertilizer. Consequently, the range of optimum fertilization levels (assuming unlimited capital) is relatively narrow, falling between about 150 and 170 pounds per acre (Figure 18) depending on the price ratio. Economic optima for situations of limited capital are not derived for this and subsequent experiments. However, recommendations made directly for "farmer consumption" might well include such information.

Comparison of the experimental results of nitrogen application on corn for Tulare County and Davis (Yolo County) indicates different response functions for the same crop in different areas. Unfortunately, this is generally the case; no single response function is applicable to a particular crop in all locations.[2/]

[1/] Unpublished data supplied by Dale G. Smeltzer, Department of Agronomy, University of California, Davis.

[2/] Conceptually, the differences due to location (soil type, temperature, etc.) could be incorporated into a "complete" production function. Empirically, however, "partial" production functions must be estimated, holding "constant" the factors due to location.

Figure 17. Corn Response to Nitrogen Fertilizer, Davis, California 1951

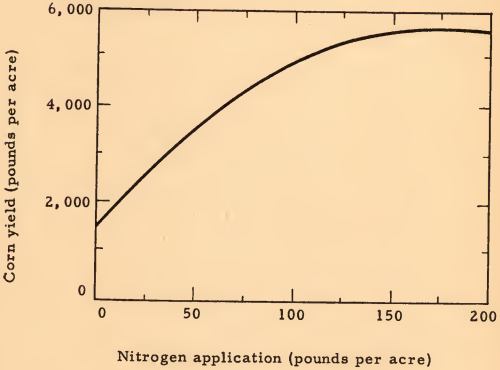
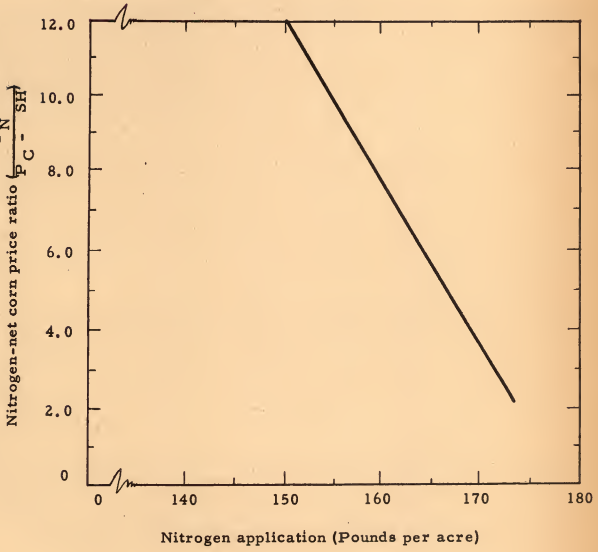


Figure 18. Relationship Between the Nitrogen-Net Corn Price Ratio and the Most Profitable Level of Application (Derived from corn response data in Figure 17)



Potatoes

Potato yield responses to fertilizers (particularly nitrogen) have been the subject of many experiments in California. Earlier experimentation was carried out by Porter and Schneider;[1] more recent work is summarized by Lorenz, et. al.[2] Potato production in the state is located mainly in Kern, Tulare, Madera and Fresno counties in the southern San Joaquin Valley. Other areas include the Tulelake district (parts of Modoc and Siskiyou counties), the Sacramento-San Joaquin Delta area and the Santa Maria Valley. Experiments in these areas indicate that potato yields respond primarily to nitrogen. Nitrogen-potato response functions for these areas are summarized in Figure 19.[3] In every case, diminishing marginal returns to nitrogen fertilizer was encountered. Substantial yield increases are apparent in all areas with nitrogen applications up to 120 pounds per acre. With the exception of the Sacramento-San Joaquin Delta area and Fresno County, the yield response curves tend to "flatten out" (incremental yield responses are less pronounced) at nitrogen applications between 120 pounds and 150 pounds per acre.

1/ Porter, D. R., "Potato Production in California," Calif. Agr. Expt. Sta. Ext. Circ. 61 (1932), and Porter, D. R. and J. B. Schneider, "Potato Production in California," Calif. Agr. Expt. Sta. Ext. Circ. 61 (1939), (revised).

2/ O. A. Lorenz, et. al., "Potato Fertilization Experiments in California," Calif. Agr. Expt. Bul. 744, 1954.

3/ The nitrogen response functions shown in Figure 19 were developed by O. A. Lorenz, et. al., from over 50 experiments conducted in commercial fields through grower cooperation with the University of California Experiment Station and Agricultural Extension Service. For further information on how the experiments were conducted see: O. A. Lorenz, et. al., *Ibid*., and "Potato Fertilizer and Blackspot Studies," Santa Maria Valley--1956, compiled by Departments of Vegetables Crops, Plant Pathology, American Potash Institute and Agr. Ext. Serv. (U. C. Series 88, January 1957).

Potatoes

Potato yield responses to fertilizing (particularly nitrogen) have been the subject of many experiments in California. Earlier experimentation was carried on by Morris and associates[1] and recent work is summarized by Lorenz, et. al.[2] Potato production in the state is located mainly in Kern, Tulare, Merced and Stanislaus counties in the southern San Joaquin Valley. Other areas include the Tulelake district [illegible] and the Siskiyou counties, the Sacramento-San Joaquin Delta area and the Santa Maria valley. Experiments in these areas indicate that potato yield respond primarily to nitrogen. [illegible] response [illegible] the experiment in Figure 13.[3] In every case, fertilizing response to nitrogen fertilizer was [illegible]. Substantial yield responses are obtained in all areas with nitrogen applications up to 100 pounds per acre. With the exception of the Sacramento-San Joaquin Delta area and Santa Maria, the yield response curves tend to flatten out (increasing yield responses are less pronounced) at nitrogen applications between 80 pounds and 150 pounds per acre.

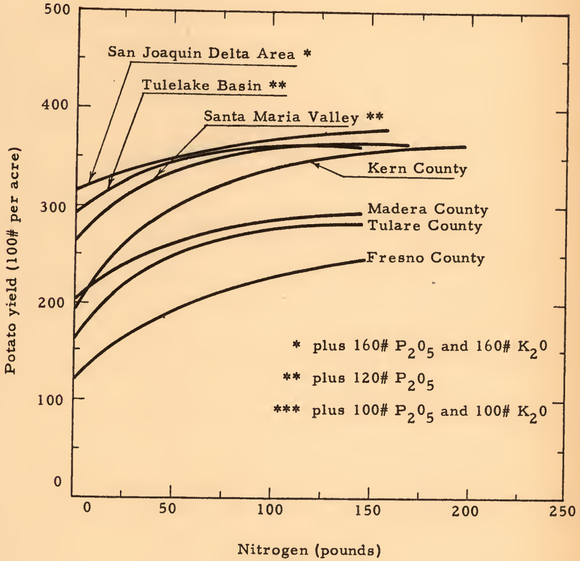
---

[1] Morris, [illegible], "Potato Production in California," Calif. Agr. Expt. Sta. Bul. 59 (1932), and [illegible] and [illegible], "Potato Production in California," Calif. Agr. Expt. Sta. [illegible] (revised).

[2] O. A. Lorenz, et. al., "Potato Fertilization Experiments in California," Calif. Agr. Expt. Sta. Bul. 744 (1954).

[3] The nitrogen response functions shown in Figure 13 were developed by O. A. Lorenz, et. al., from over 50 experiments conducted in commercial fields through joint cooperative work by the University of California Agricultural Experiment Station and Agricultural Extension Service. For further information on how the experiments were conducted see: O. A. Lorenz, et. al., op. cit., and "Potato Fertilizer and Spacing Studies, Santa Maria Valley—1955," [illegible] by Department of Vegetable Crops, Plant Pathology, [illegible] and Soil [illegible], [illegible] Series 580, January 1957.

Figure 19. Potato Yield Response to Nitrogen, Selected California Areas



Economic analyses of the physical response functions of Figure 19 use the same principles outlined earlier. One slight modification is required: Harvesting costs in potato production (as in production of many fruit, vegetable and field crops), depend on the level of yield attained. Thus, at higher levels of fertilization, "added costs" include both fertilizer costs and the additional costs associated with harvesting the increased yield.[1] Equation (11) incorporates this modification by including harvesting costs in the economic criterion.[2] In Equation (11) Y = yield of Potatoes (100 pound sacks), P_N = price per 100 pounds of nitrogen, P_p = price per 100 pounds of potatoes, H = harvest cost per 100 pounds of potatoes and

$$(11) \quad \frac{dY}{dN} = \frac{P_N}{P_p - H}$$

$\frac{dY}{dN}$ = added yield per pound of added nitrogen.[3] In effect, a "net price" for potatoes (gross price minus harvest cost per sack, $(P_p - H)$ is substituted for the "gross price" (P_p) in Equation (11) because harvest costs vary directly with yield.[4]

---

[1] The problem of including harvesting costs in potato fertilization is similar to that of including hauling and shelling costs discussed above in corn fertilization. The methods of analysis follow similar lines.

[2] There is still another consideration that should be incorporated into the economic criterion--the residual or carry-over effect. For example, the yield response on the succeeding crop (or crops) from unused nitrogen can validly be considered part of the output attributable to initial application of nitrogen. Incorporating this "residual" effect into the analysis would require, essentially, yield response functions for the residual fertilizer. Given such "residual" response functions, economic analysis would be fairly simple. Unfortunately, little is known concerning residual response. A related problem is determining optimum fertilization practices in the second and subsequent years. Economic analysis of these questions requires more extensive investigation of relationships between yield response and soil fertility.

[3] See footnote 1, page 34 for a similar derivation.

[4] An additional modification may be required where product quality, and thus realized price, is affected by level of nitrogen fertilization. Subsequent research with nitrogen application on potatoes indicates that with high levels of nitrogen (say from 150 pounds upward) the percentage of U.S. #1-size A may decrease and the percentage of #2 potatoes increase. This results from the fact that larger tubers often become growth cracked, misshapen and sun burned and are thus removed from the U.S. #1 grade. An example of the incorporation of the influence of fertilizer level on product quality is contained in the subsequent analysis of sugar beet fertilization.

[illegible] Figure 3) [illegible]

$$(14) \quad [illegible]$$

[illegible]

1/ [illegible]

2/ [illegible]

3/ [illegible]

4/ [illegible]

Table 4 summarizes the optimum level of nitrogen fertilization of potatoes for each of seven geographic areas and for a range of price-cost situations. No allowance has been made for the influence of fertilizer application level on product quality. Since potato prices fluctuate widely from year to year and fertilizer costs also change (but to a lesser extent), the nitrogen-potato price ratio shifts considerably from year to year, ranging from 4 to 33 in the six-year period 1953-1958. The optimum nitrogen levels corresponding to these ratios differ considerably by areas of the state, depending on the physical response obtained (Table 4).

Fertilization of high value crops with price uncertainty.--The potato data from Kern County illustrate a critical point in evaluating economic use of fertilizer. With high value crops, fertilizer cost is low relative to the value of the product, making relatively high levels of fertilizer profitable. Even though product prices fluctuate widely, the price ratios encourage high application levels--levels which would maximize profits when product price is high. For example, taking the 1953-1958 prices of potatoes and nitrogen, a total reduction of only about $12 in net return per acre over the six-year period would have been realized by fertilizing each year at the 200# rate rather than at the annual rate which is optimum *ex post* (Table 5). However, the optimum rate can be applied consistently only if the farmer is certain of prices and the physical response to nitrogen in each year. Since this information is unknown, the farmer must select fertilization rates based on his expectations of response and prices. Assuming he is unable to correctly anticipate prices, the producer of high value crops is often better off to fertilize at high levels (corresponding to optimum rates at favorable prices) because gains in the high-price

TABLE 4

Optimum Levels of Nitrogen Fertilization (pounds per acre) for Potatoes Grown in Seven Locations Under Different Price-Cost Situations a/

Nitrogen-potato price ratio b/ $(\frac{P_N}{P_p})$	Area						
	Kern County	Tulare County	Fresno County	Madera County	Tulelake Basin	Santa Maria Valley	San Joaquin Delta
	1	2	3	4	5	6	7
	pounds per acre						
50	110	85	115	75	55	75	20
40	125	95	140	85	60	85	65
30	135	105	--	105	70	100	105
20	150	115	--	135	80	120	130
10	200	130	--	--	105	145	--
5	--	140	--	--	130	160	--

a/ Assumes levels of P_2O_5 and K_2O at levels in Figure 19. Blanks in the table indicate that the range of nitrogen application in the experiments was not sufficiently high to estimate economic optima at these low price ratios. The optimum rate would be equal to or greater than the highest rate shown in each case.

b/ P_N = price of nitrogen per cwt.; P_p = "net price" of potatoes per cwt. where "net price" = gross price per cwt. - $0.90 per cwt. harvesting cost.

TABLE 5

Comparative Returns from Three Systems of Fertilizing Late Spring Potatoes in Kern County Over the Period 1953-1958 a/

Year	Net b/ price of potatoes ($/cwt.) 1	Price of nitrogen fertilizer ($/cwt.) 2	Nitrogen-potato price ratio (P_N/P_p) 3	Optimum nitrogen application (lbs./acre) 4	Net returns per acre (net price x yield - fertilizer costs)		
					Optimum rate 5	150# rate 6	200# rate 7
1953	$0.53	$16.75	32	135#	$ 167.13	$ 165.68	$ 161.54
1954	1.82	16.45	9	200	636.86	630.53	636.86
1955	1.00	15.85	16	175	337.26	336.22	336.30
1956	3.52	14.30	4	200	1,266.76	1,245.75	1,266.76
1957	0.43	14.10	33	135	134.91	133.65	130.04
1958	0.97	14.40	15	175	328.85	327.60	328.16
Totals					2,871.77	2,839.43	2,859.66

a/ Assuming actual and expected yields to be equal.

b/ "Net price" of potatoes = gross price per cwt. - 0.90 per cwt. harvesting costs.

years more than offset losses in the low-price years. For instance, if 150# of N were used on potatoes in each of the five years, the total return would have been reduced about $32 per acre below that corresponding to "optimum." Thus, losses from fertilizing below optimum rates were over twice as high as they would have been from fertilizing at the highest rate.

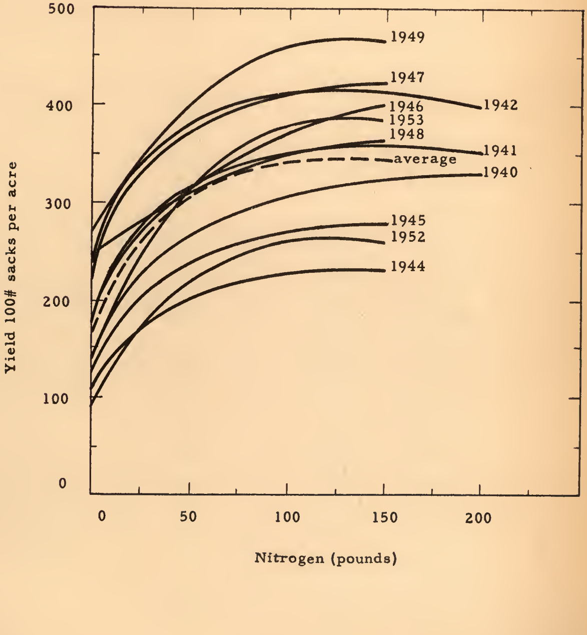
The economic importance of fertilization rates in producing high value crops can best be illustrated by considering the effects on total farm income. For a 100-acre potato enterprise, fertilizing at the 150-pound rate in a year of high potato net prices ($3.50) would "cost" the grower about $2,100 in income foregone. But, on the same farm, fertilizing at the 200-pound rate (considered optimum for high prices) would "cost" the grower only about $500 in a year of low potato net prices ($0.40). In this case, the gains from fertilizing at the heavy rate in one year of high prices more than offset four years of "losses" from heavy rates in years of low prices.

Fertilization problems with inter-year variation in response.--Experiments in Kern County on Hesperia fine sandy loam provide examples of potato yield response to applications of nitrogen.[1/] The physical response curves are shown in Figure 20, along with an "average" response curve for the two years.

In making fertilizer recommendations, two major uncertainties appear: (1) Uncertainty of future product and fertilizer prices and (2) uncertainty as to the exact nature of the physical response or production function. Assuming the physical response is known, price uncertainty can be handled in the manner illustrated above. However, in practice, the physical response function is never known exactly. The problem of handling this type of uncertainty, as illustrated below, is much more difficult.

[1/] See: O. A. Lorenz, et. al., op. cit., p. 8.

Figure 20. Potato Yield Response to Varying Levels of Nitrogen, 11 Years Period 1942 - 1953, (U.S. Cotton Field Station, Kern County)



Instability of the physical response curve is clearly illustrated in Figure 20 where the response curves shifted substantially in the eleven years shown. These response curves might be viewed as members of a "family" of such curves as discussed in the theoretical section. Ideally, potato yields would be measured in response to nitrogen alone with the effects of other factors "held constant." Apparently, the effects of some factors could not be completely "held constant" between years. Weather alone (an uncontrollable variable) might account for such variation between years. In light of such variation, what recommendation should be made? Ideally, enough experiments would be available to estimate the frequency of occurrence of the various possible functional relationships between years and for various conditions. If, for example, the 1953 nitrogen-potato response in Figure 20 occurred 9 years out of 10, recommendations based on this function alone would be quite reliable. On the other hand, if the 1953 function occurred only 4 years out of 10, the 1952 function 3 years out of 10, and the 1947 function 3 years out of 10, it becomes necessary to determine which application rate provides highest returns "on the average."1/ However, the "true" frequency of the various response functions is required for this method. Too, it may be incorrect to consider differential responses between years as random events; part of the interyear shift in the production function may be attributable to nonrandom causes which simply have not been measured.2/

1/ Unfortunately, this rate is not necessarily the optimum rate for the average function. Furthermore, the possibility of large losses or large gains in any particular year might lead "conservative" farmers or "gamblers" to rationally select fertilization rates considerably different from the rate which promises highest returns "on the average."

2/ The difficulties posed here are not unique to the "production function" approach to determining economic fertilizer usage. Explicitly or implicitly they must be recognized in any attempt to make fertilizer recommendations.

A simple alternative is the one usually employed. In this case, different experiments are pooled and treated as replications of a single experiment, and an "average" relationship derived. This "average" relationship is shown in Figure 20. Recommendations are then made from the "average" relationships.

Alfalfa

An experiment in Tulare County in 1957 on Madera loam provides an example of alfalfa yield response to applications of phosphorus.[1] The physical response curve is shown in Figure 21.

Although previous examples have considered optimum rates of nitrogen for maximum profits, the principle is the same for other nutrients (phosphorus in this case). Accordingly, Table 6 summarizes optimum rates of applying phosphate per acre for a series of phosphate-alfalfa price ratios. For example, with phosphate at $10 per hundredweight and alfalfa at a "net price" of $20 per ton ($25 minus $5 for baling and roadsiding)--a price ratio of 0.5--the most profitable rate is 130 pounds. As the price ratio decreases (the net price of alfalfa increases relative to the price of phosphate) the most profitable rate of application also increases (e.g., with a price ratio of 0.1 the optimum rate is 195 pounds). It is stressed that the above yield response to phosphate is from a single experiment in a single county and implications should not be drawn for other areas or even the same area where other growth factors may be changed. The general principle for determining the most profitable use of a nutrient--equating added returns against added costs--holds in any case.

---

[1] See: Sallee, W. R., Albert Ulrich, W. E. Martin and B. A. Krantz, "High Phosphorus for Alfalfa," California Agriculture, Vol. 13, No. 8, August 1959, page 7 to 8, 13.

Figure 21. Alfalfa Yield Response to Varying Quantities of Phosphorus (Tulare County 1957)

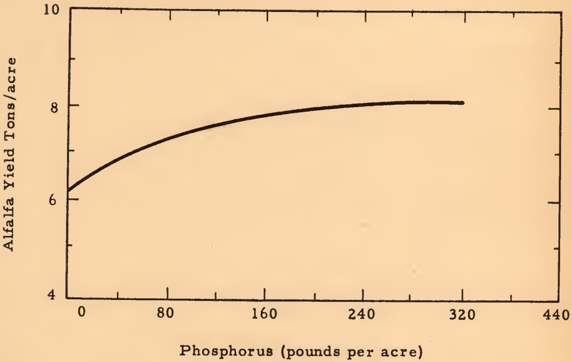


TABLE 6

Optimum Rates of Phosphate (P_2O_5) Application per Acre on Alfalfa in Tulare County (Based on 1957 Experiment on Madera Loam)

Phosphate-alfalfa price ratio a/ $(\frac{P_p}{P_a})$	Optimum rates of phosphate per acre based on: 1957 response
1	2
1.0	75
0.9	85
0.8	95
0.7	105
0.6	120
0.5	130
0.4	145
0.3	160
0.2	175
0.1	195

---

a/ The optimum rate is determined from the formula $\frac{\Delta A}{\Delta P} = \frac{P_p}{P_a}$ where ΔA = the change in alfalfa yield associated with an incremental change (ΔP) in phosphate use; P_p = price of phosphate in dollars per cwt.; P_a = "net price" of alfalfa per ton, where "net price" = "gross price" minus $5 per ton for baling and roadsiding hay for shipment.

Sugar Beets

Yield response studies for nitrogen fertilization on sugar beets have been conducted in several areas of California.[1] Figure 22 summarizes some results from these experiments. Again, diminishing physical returns is evidenced from additions of nitrogen. Sugar beets, however, present additional problems not usually encountered with other crops. Increased per acre yields of beets are usually accompanied by a decrease in average sugar content. Prices paid to growers are based both on tonnage and sugar content (i.e., total sugar produced). Hence, determination of the optimum amount of nitrogen to apply involves consideration of the quality (percent sugar) of the additional yield as well as the amount of yield.[2] Sugar beets illustrate the principles for determining economic fertilization rates when fertilization level affects quality. The principles also apply to other commodities where prices are based on quality and where a nutrient-quality relationship exists. The following analysis assumes a fixed relationship between yield level and sugar content; as more nitrogen is applied, yields rise and the percentage sugar content is assumed to fall according to this fixed relationship. An additional

---

[1] Unpublished data obtained from Mr. Jack Hills, Extension Agronomist, University of California, Davis.

[2] Prices paid to growers can not be determined directly in the case of sugar beets. Returns to growers are paid in several installments based on a price schedule set up by the sugar beet processing company. For a given yield per acre and sugar percentage, the price paid to growers is determined by the net returns to the sugar beet company from sale of the sugar. Hence, optimum fertilization rates below are based on various level of "company" net returns and nitrogen prices. The approximate prices per ton to growers corresponding to the company net returns are also given.

Sugar Beets

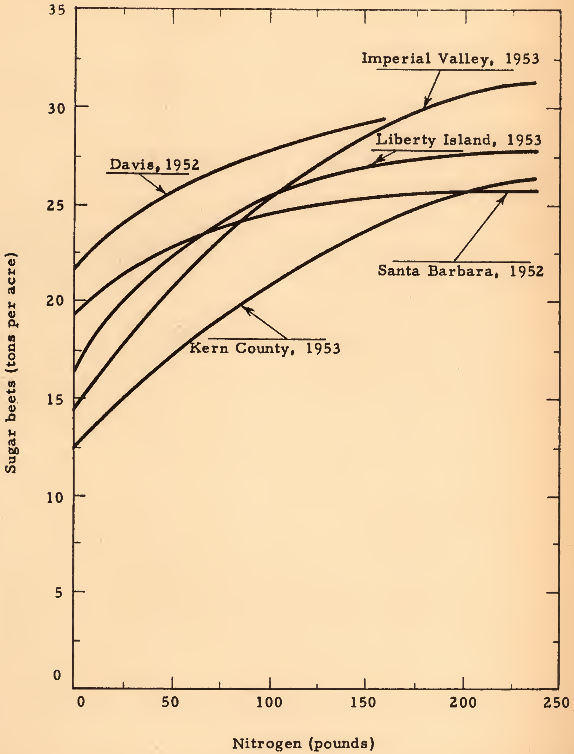
Yield response studies for nitrogen fertilization on sugar beets have been conducted in several areas of California.[1] Figure 30 summarizes the results from [illegible] Davis, [illegible] [illegible] from addition of nitrogen. Sugar beets, however, present additional problems not usually encountered with other crops. Increases in beet yields [illegible] are usually accompanied by a decrease in percentage sugar content. [illegible] determination of the optimum amount of nitrogen to apply involves consideration of the quality [illegible] of the additional yield as well as the amount of yield.[2] Sugar beets illustrate [illegible]. The principles also apply to other commodities where prices are based on quality and where a [illegible] relationship exists. The following analysis [illegible] relationships between yield [illegible] sugar content [illegible] nitrogen is applied, [illegible] and the percentage sugar content [illegible] response to [illegible] relationships [illegible]

---

[1] Unpublished data [illegible] University of California, [illegible]

[2] [illegible]

Figure 22. Sugar Beet Yield Response to Varying Quantities of Nitrogen, Selected California Areas by Years



problem is that custom harvest and hauling costs for beets are often dependent upon yield levels. All of these factors were considered in computing the most profitable nitrogen level under different price conditions. (See Appendix A for the exact procedures used.)

Table 7 summarizes briefly the most profitable rates of nitrogen fertilizer on sugar beets in five areas of California under several price-cost situations. In some areas the optimum fertilizer level depends importantly on price changes (e.g., optimum fertilizer application in Santa Barbara County ranges from 120 to 200 pounds under the various price situations). However, in other areas the range of optimum fertilization rates is quite narrow (e.g., from 200 to 240 pounds in Imperial Valley). This narrow range is determined by the nature of the physical response to nitrogen.

The price of sugar beets has been relatively stable over time, with "company net returns" in recent years around $7.00. Thus, application of nitrogen by sugar beet growers might be made with greater certainty of the economic outcomes than for some other crops.

Rice

All previous analyses have involved only a single variable nutrient. However, rice yields in the Sacramento Valley have been examined as a function of both nitrogen (N) and phosphate (P_2O_5).[1] The experimental design used for the experiment measured yield output from 10 combinations of the two fertilizer nutrients. These combinations of nutrients are plotted

---

[1] Unpublished data obtained from Dr. D. S. Mikkelsen, Agronomist, University of California, Davis.

TABLE 7

Most Profitable Range of Nitrogen Application for Sugar Beets for Five Areas of California Under Various Price-Cost Situations

Area	Company net return a/ ($)	Approximate grower price per ton b/ ($/ton)	Most profitable rates of nitrogen when price of nitrogen per cwt. is:		
			$10	$15	$20
	1	2	3	4	5
			pounds		
Santa Barbara County	$8.00	$17.85	180-200	160-180	140-160
	7.00	15.70	180-200	140-160	120-140
	6.00	13.29	180-200	140-160	120-140
Imperial Valley	$8.00	$17.54	> 240	200-220	200-220
	7.00	15.44	220-240	200-220	200-220
	6.00	13.05	200-220	200-220	200-220
Kern County	$8.00	$15.00	160-180	160-180	140-160
	7.00	13.11	160-180	140-160	140-160
	6.00	11.20	140-160	140-160	140-160
Liberty Island	$8.00	$14.85	140-160	120-140	120-140
	7.00	13.05	120-140	120-140	120-140
	6.00	11.00	120-140	120-140	100-120
Davis	$8.00	$14.14	100-120	100-120	100-120
	7.00	12.44	100-120	100-120	80-100
	6.00	10.48	100-120	80-100	60- 80

a/ Determined by sale price of sugar.

b/ Grower price differs by area depending on sugar content of beets.

in Figure 23. This "spread" of measurements allowed estimation of yield

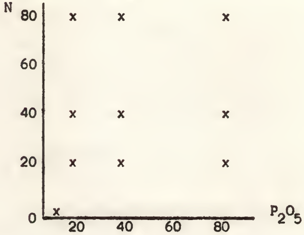


Figure 23. Combinations of Nitrogen and Phosphorus for Studying Yield Response of Rice

response to two fertilizer nutrients, giving a three-dimensional production surface. Quadratic regression Equation (12) was fitted to the observations of rice yield and combination of nitrogen and phosphorus.

$$(12)\quad R = 3127 + 42.51N - .2465N^2 + 10.91P - .0464P^2 + .1304NP$$

R = rice yield in pounds per acre, N = nitrogen in pounds applied per acre, and P = phosphate in pounds applied per acre. Equation (12) indicates diminishing returns for both nitrogen and phosphorus alone and in fixed combinations. Isoquants or "equal-yield contours" can be derived from (12) by solving the equation for N as a function of P and R* (where R* = some specified level of rice yield). This derivation is provided in Equation (13). In other words, Equation (13) can be solved for all

$$(13)\quad N = 86.22 + 26.45P \mp 2.0283\,(21.8466P - .0288P^2 + 4890 - .9860R^*)^{1/2}$$

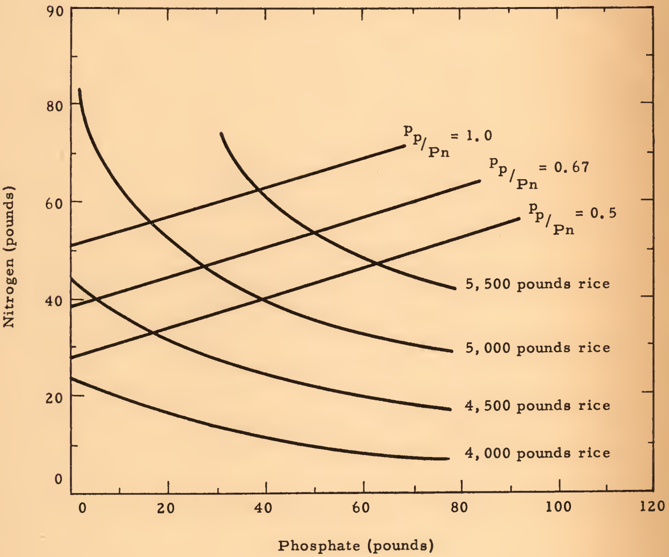
combinations of N and P which will produce some particular yield (R*). Geometrically, Equation (13) takes the form illustrated in Figure 24. The 4,000-pound isoquant shows all combinations of N and P_2O_5 which will produce a 4,000-pound yield per acre. This isoquant was obtained from Equation (13), by setting R* = 4,000 and solving for combinations of N and P which satisfy the equation. The 4,500-pound, 5,000-pound and 5,500-pound isoquant also show N and P combinations capable of producing these specified yields.

The mathematical form of (13) is such that the isoquants (curves of equal yield) are convex to the origin; i.e., over the relevant range of the variables (N and P) the slope ($^{dN}/_{dP}$) is negative. In other words, a given yield level can be attained by changing the mixture or combination of N and P applied; in this sense N and P can be called "substitutes." However, a diminishing marginal rate of substitution is found: As the quantity of nutrient 1 in the fertilizer mixture is increased relative to the quantity of nutrient 2, more and more of nutrient 1 is required to "substitute" for one unit of nutrient 2 if the same yield is to be obtained. Thus, the isoquants are curvilinear giving rise to changing marginal rates of substitution at different points on the isoquants.

The slope of the isoquant for any combination of N and P is given in Equation (14). In economic terms, the slope is the marginal rate of substitution of N for P at some given level of yield and for some combination of N and P. For example, one combination of N and P that will produce 5,000 pounds of rice is 40 pounds of N and 40 pounds of P. Substituting

$$(14) \quad \frac{dN}{dP} = \frac{10.91 - .0928P + .1304N}{42.51 - .4930N + .1304P}$$

Figure 24. Yield Isoquants and Isoclines for Selected Price Ratios, for Rice Fertilization Experiment in the Sacramento Valley



these quantities into (14) gives approximately 0.5; this indicates that at this point on the response surface one pound of P "substitutes" for 0.5 of N. Any other combination of N and P for the 5,000-pound yield level would give a different marginal rate of substitution. The least-cost combination of nutrients is at the point where the slope (or marginal rate of substitution of one nutrient for another) is equal to the inverse price ratio of the nutrients. Hence, for the price function, the minimum cost is obtained by setting Equation (14) equal to $P_p \div P_n$ as in Equation (15).

$$(15) \quad \frac{10.91 - .0928P + .1304N}{42.51 - .4930N + .1304P} = \frac{P_p}{P_n}$$

It was shown above that substituting 40 pounds of P and 40 pounds of N into the left side of Equation (15) yielded 0.5. Thus, if the phosphate-nitrogen price ratio $(P_p \div P_n) = 0.5$, then 40 pounds of P and 40 pounds of N constitute the optimum (minimum cost) combination of N and P to produce a 5,000-pound yield. It is possible to derive the equation of a line that traces out the least-cost combination of N and P for each level of rice yield for a specified price ratio. This line (isocline) is derived by setting ${}^{P}p/_{P_n} = \alpha$, then solving (15) for $N = f(P, \alpha)$ as shown in (16). Figure 24 shows three separate isoclines--for $\alpha = 1.0$,

$$(16) \quad N = \frac{P\,(.1304)\alpha + .0928P + \alpha\,42.51 - 10.91}{.4930\,\alpha + .1304}$$

0.67, and 0.5--superimposed on the yield isoquants. The intersection of the isoclines and the isoquants specify the combination of N and P that minimize costs for the respective yield level and nutrient price ratio. With a price ratio of 0.67 (e.g., the price of phosphorus is \$0.10 and the

price of nitrogen \$0.15) the least cost combination for the 4,500-pound yield level is about 38 pounds of N and 7 pounds of P; at the 5,000-pound yield level the optimum combination is 46 pounds of N and 27 pounds of P. While applications obviously are not made to such exact measurements, these amounts indicate the magnitude of changes in the fertilizer mixture necessary to minimize costs as yields change.

The location of minimum cost combinations for N and P by the above procedures can be verified or made more explicit (for those unfamiliar with the mathematical concepts) by examining costs in tabular form as shown in Table 8. Table 8 shows the combinations of N and P, estimated from Equation (13), to produce four different levels of rice yield. Also, the total cost for N and P for selected points on the isoquants are computed. From these calculations it is possible to approximately determine the combination of N and P that minimizes the cost for each yield level.1/ The minimum cost combination is where $\frac{dN}{dP} = \frac{P_p}{P_n}$.2/ For example, the minimum cost combination for a 4,500-pound yield is between P = 0, N = 43 and P = 10, N = 37 (the "exact" combination is P = 7, N = 38). For each yield level in Table 8, the point where $\frac{dN}{dP} = \frac{P_p}{P_n} = 0.67$ corresponds to the minimum total fertilizer cost for that yield level.

1/ The fact that the amounts of N and P are carried out to the exact pound does not indicate that practical recommendations to farmers can or need to be made to this degree of accuracy. The purpose of such detail here is to illustrate basic principles and to allow the reader to "work through" the equations and duplicate these figures if desired.

2/ The price ratio ($^{P_n}/_{P_p}$) exceeds the slope of the 4,000-pound yield isoquant ($^{dN}/_{dP}$) at all points, which indicates that nitrogen alone is the cheapest source of fertilizer for this case.

TABLE 8

Nutrient Combinations, Costs and Marginal Rates of Substitution for Selected Levels of Rice Yield

P (pounds)	N (pounds)	Cost of p p = $0.10	Cost of N N = $0.15	Total cost	dN/dP	P_p/P_n
1	2	3	4	5	6	7
			4,000-pound yield			
0	24	0	3.60	3.60	.46	.67
10	19	1.00	2.85	3.85	.36	.67
20	16	2.00	2.40	4.40	.31	.67
30	14	3.00	2.10	5.10	.25	.67
40	12	4.00	1.80	5.80	.21	.67
50	10	5.00	1.50	6.50	.17	.67
60	8	6.00	1.20	7.20	.14	.67
70	7	7.00	1.05	8.05	.11	.67
80	6	8.00	0.90	8.90	.09	.67
			4,500-pound yield			
0	43	0	6.45	6.45	.78	.67
10	37	1.00	5.55	6.55	.58	.67
20	31	2.00	4.65	6.65	.44	.67
30	28	3.00	4.20	7.20	.36	.67
40	24	4.00	3.60	7.60	.29	.67
50	22	5.00	3.30	8.30	.24	.67
60	19	6.00	2.85	8.85	.19	.67
70	17	7.00	2.55	9.55	.15	.67
80	16	8.00	2.40	10.40	.12	.67
			5,000-pound yield			
10	62	1.00	9.30	10.30	1.36	.67
20	52	2.00	7.80	9.80	.81	.67
30	45	3.00	6.75	9.75	.58	.67
40	40	4.00	6.00	10.00	.44	.67
50	36	5.00	5.40	10.40	.34	.67
60	33	6.00	4.95	10.95	.28	.67
70	30	7.00	4.50	11.50	.23	.67
80	28	8.00	4.20	12.20	.18	.67
			5,500-pound yield			
30	74	3.00	11.10	14.10	1.79	.67
40	62	4.00	9.30	13.30	.89	.67
50	55	5.00	8.25	13.25	.61	.67
60	49	6.00	7.35	13.35	.45	.67
70	46	7.00	6.90	13.90	.36	.67
80	42	8.00	6.30	14.30	.28	.67

[illegible]

[illegible]
[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

The preceding explanations have been concerned with locating the minimum cost combination of two nutrients at some given level of output. The next question is: What is the most profitable level of production? The determination of the isocline corresponding to specified prices of N and P gives a partial answer to this question. That is, the isocline gives the least-cost path to expand output; but which, point on the "path" gives the most profit? For the single nutrient case, the maximum profit is realized when the value of the added output is equal to the incremental cost of the input. The same condition holds for the two variable case: maximum profit is realized when the value of the added output is equal to the cost of incremental inputs combined in the least cost manner as determined by isocline Equation (16). More explicitly, the maximum profit condition for the use of N and P to produce rice is stated in Equation (17).[1]

$$(17) \quad \frac{\frac{\partial R}{\partial N} \cdot P_R}{P_N} = \frac{\frac{\partial R}{\partial P} \cdot P_R}{P_p} = 1.0$$

where $\frac{\partial R}{\partial N}$ is the change in rice yield associated with an infinitely small change in N, holding P constant (mathematically, the partial derivative of R with respect to N); $\partial R/\partial P$ is the change in rice yield associated with an infinitely small change in P, holding N constant (the partial derivative

---

[1] Equation (17) can be extended for any number of variables as follows:

$$\frac{\frac{\partial R}{\partial N_1} \; P_R}{P_{N_1}} = \frac{\frac{\partial R}{\partial N_2} \; P_R}{P_{N_2}} = \ldots. \; \frac{\frac{\partial R}{\partial N_n} \; P_R}{P_{N_n}} = 1$$

where N_1 = nutrient 1
N_2 = nutrient 2
N_n = nutrient n

of R with respect to P); P_R = price of a unit of rice and P_N and P_p are the respective price of nitrogen and phosphorus. Essentially, for known fertilizer and product prices, Equation (17) allows simultaneous solution of both the optimum combination of nutrients and the optimum level of the combination to use. For the rice data, however, no observations are available beyond 80 pounds for each nutrient and hence no recommendations as to optimum level of nutrients are made. The surface beyond 80 pounds of each is an extrapolation beyond observed data and might provide unrealistic results.

EXPERIMENTAL DESIGNS FOR GENERATING THE YIELD RESPONSE SURFACE

Emphasis above has been on methods, procedures and logic for determining economic rates of fertilizer use. This section briefly examines possible experimental procedures which would generate data amenable to economic analysis.

This study has been based primarily on experiments measuring yield response to a single nutrient--nitrogen in most cases. These experiments require successively higher nutrient levels ranging from zero to a magnitude sufficient to encounter diminishing total yield. Other variables are, of course, held at some constant level or are assumed to vary in a random manner allowing appropriate probability statements to be made regarding their effects.1/ Since the objective in this case is to determine a continuous response curve, it is important to have more levels of treatment

---

1/ In order to make statistical inferences from the data, it is required that treatments be assigned to plots in some random manner (e.g., complete random design, random block, etc.).

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

than is required when the objective is to test for treatment effects.[1] Furthermore, some of the treatment levels should be beyond the range which is considered a priori economically feasible. These levels provide an accurate measurement of the response function at the point where marginal returns are nearing zero. In many conventional experiments, the highest level of treatment still shows substantial marginal yield increases, indicating that "optimum" fertilization rates are beyond the range of observed data. Lacking observations at these heavy rates, an "optimum" rate can be estimated only by dangerous extrapolation beyond observed data. To compensate for the additional treatments required to estimate a continuous function, the number of replications per treatment may be reduced. Also, replications need not be equal for each treatment; fewer points are needed over the "expected" uneconomical ranges than over the relevant economic range. These ranges should be approximately established from previous experiments and knowledge of the crop involved. Table 9 presents an example of one possible arrangement of treatments and replications for a nitrogen experiment on sugar beets. Nine levels of nitrogen are used; four replications are made for 0 (in order to estimate accurately the yield intercept on the response curve), 100, 125, 150, 175 and 200-pound levels; two

---

1/ Any continuous function derived will vary from year to year in shape and slope depending on soil conditions, residual fertilizer in the soil, rainfall, etc. These conditions are not unique for this type of analysis but rather are common to any experimental procedure. Certainly, further experimental work should be directed toward more accurate measurement of the effects of these "fixed" and "random" factors. As a matter of practice, however, experiments are usually conducted on land tracts considered to be "typical" for the area in terms of nutrient content, soil structure and other conditions.

replications are made for 50, 250 and 300-pound levels. In total, 30 plots are required compared to the usual four treatments with six replications requiring 24 plots. The value of the additional treatment levels in the former experiment is in attaining a more complete approximation to the overall response curve rather than added precision for a few selected points on the curve.

TABLE 9

Example of Data Arrangement for Determining Single Variable Response Curve for Sugar Beets

Pounds of nitrogen per acre	Number of replications
1	2
0	4
50	2
100	4
125	4
150	4
175	4
200	4
250	2
300	2

Experiments for Two or More Nutrients

Agronomic and economic consideration of joint and interaction effects of more than one variable requires estimation of a yield response surface. Since the response surface is logically the extension of a response curve into multidimensional space, the data arrangement or design to approximate the surface is an extension of the design required for the response curve. Accordingly, one possible arrangement of treatments for estimating the

response surface is a complete factorial (i.e., all combinations of treatments for each nutrient), However, complete factorial arrangement for a sufficient number of levels with replications to estimate a response surface requires an increasingly large number of plots. The required plots increase at the rate of $r\ T^v$, where r = replications, v = number of variables, and T = number of treatments. For example, with 2 nutrients, 9 levels of each nutrient replicated twice would require 162 plots ($2 \times 9^2 = 162$). Of course, it is possible to reduce the total number of plots by excluding certain nutrient combinations over the surface. Table 10 is an example of an experiment where treatments are arranged factorially but with various combinations excluded. This particular arrangement has been used extensively on corn fertilization in Iowa.1/ Again it should be pointed out that the reasoning behind the wide range of treatments with fewer replications is that yield may be approximated by a continuous function when the level of N and P are varied simultaneously.2/

1/ Heady, E. O., J. Pesek and W. Brown, op. cit., and Heady, E. O., and John Pesek, "A Fertilizer Production Surface," Journal of Farm Economics, Vol. 36, August 1954, pp. 453-582.

2/ Recently, Box and others have introduced designs expressly for purposes of fitting second order polynomials. These designs require fewer treatment combinations than are required for the complete factorial arrangement. Moreover, they are considered to be equally or more efficient on a per unit observation basis for estimating parameters in second order polynomial equations than factorial or even fractional factorials. For additional information on construction and advantages of these composite designs see: Box, G.E.P., and K. B. Wilson, "On the Experimental Attainment of Optimum Conditions," Journal of Royal Soc., Series B XIII, No. 1, 1951; Box and Wilson, "The Exploration and Exploitation of Response Surfaces," Biometrics, 10:16-60, 1954; Hader, R. J., M. E. Harward, D. D. Mason and D. P. Moore, "An Investigation of Some of the Relationships of Copper, Iron, and Molybdenum in the Growth and Nutrition of Lettuce: I Experimental Designs and Statistical Methods for Characterizing the Response Surface," Soil Sci. Soc. Amer. Proc., 21:59-64, 1957. Also see: Tramel, T. E., "A Suggested Procedure for Agronomic-Economic Fertilizer Experiments," Fertilizer Innovations and Resource Use, edited by E. L. Baum, et. al., Iowa State Press, Chapter 15, 1957.

TABLE 10

Example of Experimental Arrangement for N
and P to Estimate a Response Surface
(Figures within squares designate number of replications)

Pounds P_2O_5 per acre	Pounds nitrogen per acre								
	0	40	80	120	160	200	240	280	320
	1	2	3	4	5	6	7	8	9
0	2	2	2	2	2	2	2	2	2
40	2	2			2	2			2
80	2		2		2		2		2
120	2			2	2			2	2
160	2	2	2	2	2	2	2	2	2
200	2	2			2	2			2
240	2		2		2		2		2
280	2			2	2			2	2
320	2	2	2	2	2	2	2	2	2

APPENDIX A

In the text, mathematical procedures have been set forth for locating the optimum level of fertilization under the usual conditions of a constant price per unit of product. Following is a specific example of determining optimum fertilization levels with the assumption of constant product prices relaxed. The pricing of sugar beets is an example of the latter condition where the price per ton is dependent upon the percent sugar content. The following derivation is based on the assumption that the percent sugar content is a function of the nitrogen level. While a simple linear relationship between nitrogen and sugar percentage is hypothesized here, further experimental evidence may indicate more complex relationships which would have to be incorporated into the analysis.

The following terms are defined:

π = profit per acre

Y = yield of sugar beets per acre in tons

R = return per ton of sugar beets

N = pounds nitrogen applied per acre

X = percentage sugar content

V.C. = variable costs per acre

F.C. = fixed costs

H = harvest and hauling cost per ton

P_n = nitrogen cost per pound

Hence, the standard profit equation is written as Equation (a).

(a) $\pi = YR - \text{V.C.} - \text{F.C.}$

In this case, additional relationships among the variables are noted in Equations (b), (c) and (d).

(b) $R = f(X)$

(c) $X = f(N)$

(d) $V.C. = H \cdot Y + N \cdot P_n$

To find the level of nitrogen (N) associated with maximum profits (π), take the derivative of Equation (a) with respect to N and set the result equal to zero, as in Equation (e). Equation (f) gives the condition for maximum profits.

(e) $$\frac{\partial \pi}{\partial N} = \frac{\partial Y}{\partial N} R + \frac{\partial R}{\partial N} Y - H \frac{\partial Y}{\partial N} - P_n = 0$$

(f) $$\frac{\partial Y}{\partial N} = \frac{P_n - \frac{\partial R}{\partial N} Y}{R - H}$$

Example:

A yield response function for sugar beets in Santa Barbara County is given in (g); the marginal response function is given in (h).

(g) $Y = 19.73 + .0628N - .00016N^2$

(h) $\frac{dY}{dN} = .0628 - .00032N$

The revenue function for the grower corresponding to the $7.00 company net return level is given in (i)

(i) $R = - .59 + .90X$

The relationship between percentage sugar content (X) and nitrogen was estimated from similar experimental data used to estimate (h) and is given in (j) and (k).

(j) $X = 18.6 - .00125N$

(k) $\frac{\partial X}{\partial N} = -.00125N$

Harvest and hauling costs are approximately \$2.50 per ton. Accordingly, the π equation can be rewritten as shown in (1)

(1) $\pi = Y\,(-.59 + .90X) - Y\,(2.50) - NP_n$

The derivative of π with respect to N is given in (m)

(m) $\frac{\partial \pi}{\partial N} = \frac{\partial Y}{\partial N}\,(-3.09 + .90X) + Y\,\frac{\partial X}{\partial N}\,(.90) - P_n$

Substituting equations (g), (h), (j) and (k) into (m), setting the result equal to zero, and solving for N gives:

(n) $N = \frac{.44457 - \sqrt{.19042925 + .00864P_n}}{.432 \times 10^{-4}}$

Equation (n) defines the most profitable level of N to apply for any value of P_n when payments to growers are based on a \$7.00 "net return" for the company. A corresponding equation could be derived for any specified company net return. As the price of nitrogen (P_n) increases, equation (n) indicates that the most profitable level of N to apply decreases. For example, with P_n = \$0.10 per pound, the most profitable level of N is 167 pounds; with P_n = \$0.15 per pound, the optimum N = 155 pounds; with P_n = \$0.20 per pound the optimum level of N is 144 pounds.